MATHEMATICS AND STATISTICS (Div III)
MATHEMATICS
Chair: C. Carlisle and Margaret Tippit Professor of Statistics, Richard De Veaux

- Colin C. Adams, Thomas T. Read Professor of Mathematics
- Palak Arora, Visiting Assistant Professor of Mathematics
- Christina Athanasouli, Assistant Professor of Mathematics
- Bhagya Athukorallage, Visiting Assistant Professor of Mathematics
- Julie C. Blackwood, Associate Professor of Mathematics
- Xizhen Cai, Assistant Professor of Statistics
- Duncan A. Clark, Assistant Professor of Statistics
- Daniel Condon, Visiting Assistant Professor of Mathematics
- Richard D. De Veaux, C. Carlisle and Margaret Tippit Professor of Statistics
- Thomas A. Garrity, Webster Atwell Class of 1921 Professor of Mathematics
- Leo Goldmakher, Associate Professor of Mathematics
- Pamela E. Harris, Associate Professor of Mathematics
- Stewart D. Johnson, Professor of Mathematics
- Susan R. Loepp, William R. Kenan, Jr. Professor of Mathematics
- Steven J. Miller, Professor of Mathematics
- Ralph E. Morrison, Associate Professor of Mathematics
- Anna C. Neufeld, Assistant Professor of Statistics
- Shaoyang Ning, Assistant Professor of Statistics
- Allison Pacelli, Professor of Mathematics
- Lori A. Pedersen, Lecturer in Mathematics
- Anna M. Plantinga, Assistant Professor of Statistics
- Norean R. Sharpe, Visiting Professor of Statistics
- Cesar E. Silva, Chair & Hagey Family Professor of Mathematics
- Mihai Stoiciu, Professor of Mathematics
- Ivo Terek, Visiting Assistant Professor of Mathematics
- Elizabeth M. Upton, Assistant Professor of Statistics

MAJOR IN MATHEMATICS
The major in Mathematics is designed to meet two goals: to introduce some of the central ideas in a variety of areas of mathematics and statistics, and to develop problem-solving ability by teaching students to combine creative thinking with rigorous reasoning. Mathematics is a gateway to many career paths including statistics, teaching, consulting, business, engineering, finance, actuarial studies and applied mathematics. Students are strongly encouraged to consult with the department faculty on choosing courses appropriate to an individualized program of study.

REQUIREMENTS (9 courses plus colloquium)
The major in Mathematics consists of nine courses taken at Williams plus the colloquium requirement. Mathematics is highly cumulative, and students should plan a route to completing the major that ensures the proper sequencing and prerequisites for all needed courses. Note that not all upper level courses are offered every year.
Calculus (2 courses)
Mathematics 140 Calculus II
Mathematics 150 or 151 Multivariable Calculus

Applied/Discrete Mathematics/Statistics (1 course)
or Mathematics 210 Mathematical Methods for Scientists (Same as Physics 210)
or Mathematics 200 Discrete Mathematics
or Statistics 201 Statistics and Data Analysis
Mathematics 309 Differential Equations
or a more advanced applied/discrete/statistics course with prior department approval

Core Courses (3 courses)
Mathematics 250 Linear Algebra
Mathematics 350 Real Analysis or Mathematics 351 Applied Real Analysis
Mathematics 355 Abstract Algebra

Completion (3 courses plus colloquium)
Two mathematics or statistics electives from courses numbered 300 and above
One Senior Seminar: Any mathematics or statistics course numbered between 400 and 479, taken at Williams College in the junior or senior year.
Participation in the Department Colloquium, in which all senior majors present a talk on a mathematical or statistical topic of their choice. Each major must attend at least 20 colloquia (reduced to 15 during the Academic Year 2020-2021), and up to 5 attendances may be counted in their junior year. Students engaged in study away may petition the department in advance to count up to 5 suitable colloquia attendances from their study away program.

Pass/Fail policies during the Academic Year 2020-2021
Information about the Department of Mathematics and Statistics Pass/Fail policies during the Academic Year 2020-2021 can be found here.

ADVANCED PLACEMENT
Students who come to Williams with advanced placement will be moved up in the Mathematics major, and should consult with faculty to be placed in the best class reflecting their experience and background. A student who places out of a course substitutes another course of equal or higher level and numbered 200 or above in Mathematics or Statistics to complete the nine course major. Students should select courses best suited to their preparation and goals, and consult with the department faculty concerning appropriate courses and placement. The department reserves the right to refuse registration in any course for which the student is overqualified.

For Example, a student starting in MATH 130 might take MATH 130 and 140 the first year, MATH 150 and MATH 200 the second year, MATH 250 and MATH 350 the third year, MATH 355 and a senior seminar the fourth year, plus the two required electives some time. Students are encouraged to consult freely with any math faculty about course selection and anything else.

CALCULUS PLACEMENT
Recommended placement for students who have taken an Advanced Placement Examination in Calculus (AB or BC) is:
BC 1, 2 or AB 2, 3 Math 140
AB 4 or 5 Math 150
BC 3, 4 or 5 Math 151

Consult with department faculty for any Calculus or Statistics placement questions. Students who have had calculus in high school, whether or not they took the Advanced Placement Examination, are barred from 130 unless they obtain permission from the instructor.

NOTES
Substitutions, Study Abroad, and Transfer Credit: In some cases, and with prior permission of the Mathematics and Statistics Department, appropriate courses from other institutions or a course from another Williams department may be substituted for electives. Programs like the “Budapest Semester in Mathematics” are recommended for majors who wish to focus on mathematics away. The department, though, normally
accommodates students who select other study away programs. The department offers its core courses in both the fall and the spring to allow students to spend more easily a semester away. The senior seminar requirement is expected to be fulfilled at Williams College.

**Double Counting:** No course may count towards two different majors.

**Planning Courses:** Core courses Mathematics 350/351 and 355, are normally offered every year. Most other 300-level topics are offered in alternate years. Topology, Complex Analysis, and second courses in real analysis and abstract algebra are normally offered at least every other year.

Each 400-level topic is normally offered every two to four years. Students should check with the department before planning far into the future.

**Course Admission:** Courses are normally open to all students meeting the prerequisites, subject to any course caps. Students with questions about the level at which courses are conducted are invited to consult department faculty.

**FAQ**

Students MUST contact departments/programs BEFORE assuming study away credit will be granted toward the major or concentration.

Can your department or program typically pre-approve courses for major/concentration credit?

Yes, in many cases, though students should be sure to contact the department.

What criteria will typically be used/required to determine whether a student may receive major/concentration credit for a course taken while on study away?

Course title and description, and complete syllabus including readings/assignments.

Does your department/program place restrictions on the number of major/concentration credits that a student might earn through study away?

No.

Does your department/program place restrictions on the types of courses that can be awarded credit towards your major?

Yes. They have to be approved MATH/STAT courses.

Are there specific major requirements that cannot be fulfilled while on study away?

Yes. Colloquium requirement, Senior Seminar requirement.

Are there specific major requirements in your department/program that students should be particularly aware of when weighing study away options? (Some examples might include a required course that is always taught in one semester, laboratory requirements.)

Yes. The highly cumulative structure of the major.

Give examples in which students thought or assumed that courses taken away would count toward the major or concentration and then learned they wouldn’t:

None to date.

**THE DEGREE WITH HONORS IN MATHEMATICS**

The degree with honors in Mathematics is awarded to the student who has demonstrated outstanding intellectual achievement in a program of study which extends beyond the requirements of the major. The principal considerations for recommending a student for the degree with honors will be: Mastery of core material and skills, breadth and, particularly, depth of knowledge beyond the core material, ability to pursue independent study of mathematics or statistics, originality in methods of investigation, and, where appropriate, creativity in research.

An honors program normally consists of two semesters (MATH/STAT 493 and 494) and a winter study (WSP 031) of independent research, culminating in a thesis and a presentation. Under certain circumstances, the honors work can consist of coordinated study involving a one semester (MATH/STAT 493 or 494) and a winter study (WSP 030) of independent research, culminating in a “minithesis” and a presentation. Note that during the Academic Year 2020-2021 the winter study requirement for thesis and “minithesis” is waived. At least one semester should be in addition to the major requirements, and thesis courses do not count as 400-level senior seminars.

An honors program in actuarial studies requires significant achievement on four appropriate examinations of the Society of Actuaries.

Highest honors will be reserved for the rare student who has displayed exceptional ability, achievement or originality. Such a student usually will have written a thesis, or pursued actuarial honors and written a mini-thesis. An outstanding student who writes a mini-thesis, or pursues actuarial honors and writes a paper, might also be considered. In all cases, the award of honors and highest honors is the decision of the Department.

**APPLIED MATHEMATICS TRACK**

Students interested in applied mathematics, engineering, or other sciences should consider:

Mathematics 140 Calculus II

Mathematics 150 or 151 Multivariable Calculus
Statistics 201 Statistics and Data Analysis
Mathematics 250 Linear Algebra
Mathematics 351 Applied Real Analysis
Mathematics 355 Abstract Algebra
Some programming or numerical analysis (e.g. MATH 361, 318T, or anything if you’ve had CSCI 134)
MATH 309 or Post-core Differential Equations/Numerical Methods
Senior seminar (e.g. Math Ecology MATH 410T or Mathematical Modeling MATH 433)

Other recommended courses: complex analysis, discrete mathematics (e.g. combinatorics or graph theory), operations research, optimization, probability, statistics, appropriate courses in Biology, Chemistry, Computer Science, Economics, Neuroscience, Physics, etc.

Williams has exchange and joint programs with good engineering schools. Interested students should consult the section on engineering near the beginning of the Bulletin and the Williams pre-engineering advisor for further information.

GRADUATE SCHOOL TRACK
Students interested in continuing their study of mathematics in graduate school should consider:

Mathematics 140 Calculus II
Mathematics 150 or 151 Multivariable Calculus
Mathematics 250 Linear Algebra
Mathematics 350 Real Analysis
Mathematics 355 Abstract Algebra
Complex Analysis
Topology
Some second semester analysis
Some second semester algebra
Some post-core geometry
Thesis

[With prior permission, in unusual circumstances, senior seminar can be waived in favor of harder post-core electives.]

Students headed for graduate school generally take more than this relatively small number of courses required for a liberal arts major. Reading knowledge of a foreign language (French, German, or Russian) can be helpful.

Students interested in studying statistics in graduate school should take STAT 201, 346, 360, a 400 level statistics course and MATH 350/351 and 341 in addition to their other math requirements.

OTHER CAREER PATHS

Other Graduate and Professional Schools: An increasing number of graduate and professional schools require mathematics and statistics as a prerequisite to admission or to attaining their degree. Students interested in graduate or professional training in business, medicine, economics, or psychology are advised to find out the requirements in those fields early in their college careers.

Statistics and Actuarial Science: Students interested in statistics or actuarial science should consider Mathematics 341, Statistics courses, and Economics 255. Additionally, students should consider taking some number of the actuarial exams given by the Society of Actuaries, which can constitute part of an honors program in actuarial studies (see section on honors above).

Teaching: Students interested in teaching mathematics at the elementary or secondary school level should consider courses on teaching, number theory, geometry, statistics, and practice as a tutor or teaching assistant. Winter study courses that provide a teaching practicum are also highly recommended. Consult the Program in Teaching (Professor Susan Engel) and the Office of Career Counseling.

Business and Finance: Students interested in careers in business or finance should consider Mathematics 333 and Statistics courses. Since these courses address different needs, students should consult with the instructors to determine which seem to be most appropriate for individuals.

There are three types of 300-level courses. There are the core courses: Real Analysis, MATH 350/351, and Abstract Algebra Math 355. There are
the “precore” courses, which do not have the core courses as prerequisites and have numbers 300-349. Finally, there are those courses that have an Abstract Algebra or Real Analysis prerequisite, which are numbered 360-399.

MATH 102 (F) Foundations in Quantitative Skills
This course will strengthen a student's foundation in quantitative reasoning in preparation for the science curriculum and QFR requirements. The material will be at the college algebra/precalculus level, and covered in a tutorial format with students working in small groups with the professor. Access to this course is limited to placement by a quantitative skills counselor.

Requirements/Evaluation: homework, presentations during the tutorial meetings, and projects
Prerequisites: access to the course is limited to placement by a quantitative skills counselor
Enrollment Limit: 10
Enrollment Preferences: students who need most help with the quantitative reasoning
Expected Class Size: 10
Grading: no pass/fail option, no fifth course option
Distributions: (D3)

Fall 2024
TUT Section: T1 TBA Julie C. Blackwood

MATH 113 (S) The Beauty of Numbers (QFR)
This course will be an introduction to number theory and mathematical thinking and logic, with emphasis throughout on mathematics as a way of thinking and approaching the world. Have you ever wondered what keeps your credit card information safe every time you buy something online? Number theory! Number Theory is one of the oldest branches of mathematics. In this course, we will discover the beauty and usefulness of numbers, from ancient Greece to modern cryptography. We will look for patterns, make conjectures, and learn how to prove these conjectures. Starting with nothing more than basic high school algebra, we will develop the logic and critical thinking skills required to realize and prove mathematical results. Topics to be covered include the meaning and content of proof, prime numbers, divisibility, rationality, modular arithmetic, Fermat's Last Theorem, the Golden ratio, Fibonacci numbers, coding theory, and unique factorization. This course is meant to give you an appreciation for numbers and mathematics and to enhance your logical reasoning skills. Although most people will not use calculus or geometry in their jobs or everyday lives, mathematics enhances our abilities to think logically and reason effectively. This skill is useful in all aspects of life. Number theory, in particular, is a great area of mathematics that allows one to jump in right away without a lot of pre-requisite knowledge. We will look at examples, look for patterns, make conjectures, and we will spend a lot of time learning how to rigorously prove those conjectures.

Requirements/Evaluation: projects, homework assignments, and exams
Prerequisites: MATH 102 (or demonstrated proficiency on a diagnostic test) or permission of instructor. Anyone who has previously taken a 200-level math course or higher must obtain instructor permission to take the course.
Enrollment Limit: 25
Enrollment Preferences: If over-enrolled, course selection will be based on answers to a questionnaire.
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: proof writing and logic

Spring 2025
LEC Section: 01 TR 11:20 am - 12:35 pm Allison Pacelli

MATH 130 (F)(S) Calculus I (QFR)
Calculus permits the computation of velocities and other instantaneous rates of change by a limiting process called differentiation. The same process
also solves "max-min" problems: how to maximize profit or minimize pollution. A second limiting process, called integration, permits the computation of areas and accumulations of income or medicines. The Fundamental Theorem of Calculus provides a useful and surprising link between the two processes. Subtopics include trigonometry, exponential growth, and logarithms.

Requirements/Evaluation: Weekly homework and quizzes, 2 exams during the semester, and one final

Prerequisites: MATH 102 (or demonstrated proficiency on a diagnostic test); this is an introductory course for students who have not seen calculus before

Enrollment Limit: 50

Enrollment Preferences: first-year students

Expected Class Size: 20

Grading: yes pass/fail option, yes fifth course option

Unit Notes: students who have previously taken a calculus course may not enroll in MATH 130 without the permission of instructor

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This a calculus course.

Fall 2024
LEC Section: 01    MWF 8:00 am - 8:50 am    Lori A. Pedersen
LEC Section: 02    MWF 9:00 am - 9:50 am    Lori A. Pedersen

Spring 2025
LEC Section: 01    MWF 9:00 am - 9:50 am    Leo Goldmakher

MATH 140 (F)(S) Calculus II (QFR)

Calculus answers two basic questions: how fast is something changing (the derivative) and how much is there (the integral). This course is about integration, and the miracle that unites the derivative and the integral (the Fundamental Theorem of Calculus.) Understanding calculus requires in part the understanding of methods of integration. This course will also solve equations involving derivatives ("differential equations") for population growth or pollution levels. Exponential and logarithmic functions and trigonometric and inverse functions will also play an important role. This course is the right starting point for students who have seen derivatives, but not necessarily integrals, before.

Requirements/Evaluation: homework, quizzes, and/or exams

Prerequisites: MATH 130 or equivalent; students who have received the equivalent of advanced placement of AB 4, BC 3 or higher may not enroll in MATH 140 without the permission of instructor

Enrollment Limit: 50

Enrollment Preferences: based on who needs calculus the soonest

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Unit Notes: students who have higher advanced placement must enroll in MATH 150 or above

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This a math class

Fall 2024
LEC Section: 01    MWF 9:00 am - 9:50 am    Bhagya Athukorallage
LEC Section: 02    MWF 12:00 pm - 12:50 pm    Bhagya Athukorallage

Spring 2025
LEC Section: 01    MWF 10:00 am - 10:50 am    Christina Athanasouli

MATH 150 (F)(S) Multivariable Calculus (QFR)

Applications of calculus in mathematics, science, economics, psychology, the social sciences, involve several variables. This course extends calculus to several variables: vectors, partial derivatives, multiple integrals. There is also a unit on infinite series, sometimes with applications to differential
equations.

**Requirements/Evaluation:** Problem sets and exams

**Prerequisites:** MATH 140 or equivalent, such as satisfactory performance on an Advanced Placement Examination

**Enrollment Limit:** 50

**Enrollment Preferences:** Preference will be given to prospective math and stats majors, or students who need this as a course to serve as a prerequisite for other courses.

**Expected Class Size:** 40

**Grading:** yes pass/fail option, yes fifth course option

**Unit Notes:** Students with the equivalent of advanced placement of AB 4 or above should enroll in MATH 150, students with a BC 3 or higher should enroll in Math 151 when it is being offered, and Math 150 otherwise.

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** mathematics

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**Fall 2024**

LEC Section: 01  MWF 9:00 am - 9:50 am  Ralph E. Morrison
LEC Section: 02  MWF 10:00 am - 10:50 am  Ralph E. Morrison
LEC Section: 03  MWF 12:00 pm - 12:50 pm  Ralph E. Morrison

**Spring 2025**

LEC Section: 01  MWF 10:00 am - 10:50 am  Palak Arora

**MATH 151 (F) Multivariable Calculus** (QFR)

Applications of calculus in mathematics, science, economics, psychology, the social sciences, involve several variables. This course extends calculus to several variables: vectors, partial derivatives and multiple integrals. The goal of the course is Stokes Theorem, a deep and profound generalization of the Fundamental Theorem of Calculus. The difference between this course and MATH 150 is that MATH 150 covers infinite series instead of the theorems of vector calculus. Students with the equivalent of BC 3 or higher should enroll in MATH 151, as well as students who have taken the equivalent of an integral calculus and who have already been exposed to infinite series. For further clarification as to whether MATH 150 or MATH 151 is appropriate, please consult a member of the math/stat department.

**Requirements/Evaluation:** problem sets and exams

**Prerequisites:** AP BC 3 or higher or integral calculus with infinite series

**Enrollment Limit:** 50

**Enrollment Preferences:** First-years, sophomores, and juniors

**Expected Class Size:** 40

**Grading:** yes pass/fail option, yes fifth course option

**Unit Notes:** MATH 151 satisfies any MATH 150 prerequisite; credit will not be given for both MATH 150 and MATH 151

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This course builds quantitative skills

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**Fall 2024**

LEC Section: 01  MWF 9:00 am - 9:50 am  Susan R. Loepp
LEC Section: 02  MWF 10:00 am - 10:50 am  Susan R. Loepp
LEC Section: 03  MWF 11:00 am - 11:50 am  Susan R. Loepp

**MATH 197 (F) Independent Study: Mathematics**

Directed 100-level independent study in Mathematics.

**Requirements/Evaluation:** decided by the department
MATH 198  (S)  Independent Study: Mathematics
Directed 100-level independent study in Mathematics.
Prerequisites: permission of the department
Enrollment Limit: 20
Enrollment Preferences: decided by the department
Expected Class Size: 1
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)
Not offered current academic year

MATH 200  (F)(S)  Discrete Mathematics  (QFR)
In contrast to calculus, which is the study of continuous processes, this course examines the structure and properties of finite sets. Topics to be covered include mathematical logic, elementary number theory, mathematical induction, set theory, functions, relations, elementary combinatorics and probability, and graphs. Emphasis will be given on the methods and styles of mathematical proofs, in order to prepare the students for more advanced math courses.
Class Format: This will not be a typical lecture course; instead it will be a blend of lecture and discovery-based learning, with weekly small group meetings with TA's.
Requirements/Evaluation: Spring: Evaluation will be based on homework, exams, and participation in weekly small group meetings.
Prerequisites: Calculus at the level of an AP course or Williams College Math 130 or 140. Students who have taken a 300-level or 400-level math course should obtain permission of the instructor before enrolling.
Enrollment Limit: 40
Enrollment Preferences: Preference given to first and second year students intending to major in mathematics or computer science.
Expected Class Size: 40
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This course involves developing the formal mathematical language of logic and set theory. It also involves using quantitative tools to solve problems relating to combinatorics, probability, number theory, and other fields of discrete mathematics.

Fall 2024
LEC Section: 01  TR 9:55 am - 11:10 am  Palak Arora
LEC Section: 02  TR 11:20 am - 12:35 pm  Palak Arora

Spring 2025
LEC Section: 01  TR 8:30 am - 9:45 am  Lori A. Pedersen
LEC Section: 02  TR 9:55 am - 11:10 am  Lori A. Pedersen

MATH 210  (S)  Mathematical Methods for Scientists  (QFR)
This course covers a variety of mathematical methods used in the sciences, focusing particularly on the solution of ordinary and partial differential equations. In addition to calling attention to certain special equations that arise frequently in the study of waves and diffusion, we develop general techniques such as looking for series solutions and, in the case of nonlinear equations, using phase portraits and linearizing around fixed points. We study some simple numerical techniques for solving differential equations. An optional session in Mathematica will be offered for students who are not already familiar with this computational tool.

**Class Format:** three hours per week

**Requirements/Evaluation:** several exams and weekly problem sets, all of which have a substantial quantitative component

**Prerequisites:** MATH 150 or 151 and familiarity with Newtonian mechanics at the level of PHYS 131

**Enrollment Limit:** 50

**Enrollment Preferences:** sophomores and juniors

**Expected Class Size:** 30

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:

PHYS 210(D3) MATH 210(D3)

**Quantitative/Formal Reasoning Notes:** This course will have weekly problem sets using advanced calculus methods
MATH 297 (F) Independent Study: Mathematics
Directed 200-level independent study in Mathematics.

Requirements/Evaluation: decided by the department
Prerequisites: permission of department
Enrollment Limit: 20
Enrollment Preferences: decided by the department
Expected Class Size: 1
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)
Not offered current academic year

MATH 298 (S) Independent Study: Mathematics
Directed 200-level independent study in Mathematics.

Requirements/Evaluation: decided by the department
Prerequisites: permission of department
Enrollment Limit: 20
Enrollment Preferences: decided by the department
Expected Class Size: 1
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)
Not offered current academic year

MATH 307 (S) Computational Linear Algebra (QFR)
Linear algebra is of central importance in the quantitative sciences, including application areas such as image and signal processing, data mining, computational finance, structural biology, and much more. When the problems must be solved computationally, approximation, round-off errors, convergence, and efficiency matter, and traditional linear algebra techniques may fail to succeed. We will adopt linear algebra techniques on a large scale, implement them computationally, and apply them to core problems in scientific computing. Topics may include: systems of linear and nonlinear equations; approximation and statistical function estimation; optimization; interpolation; data scraping; singular value decomposition; and more. This course could also be considered a course in numerical analysis or computational science.

Class Format: This course is taught in a flipped classroom format. Students read and watch lecture videos prior to each class session. The instructor uses class time for discussion and collaborative learning activities. This course will be a good fit for students with a strong interest in applied mathematics and a willingness to devote significant effort to learning/doing computer programming.

Requirements/Evaluation: Students will complete regularly assigned activities and problem sets. To move towards a non-hierarchical, transparent, and egalitarian grading system, the instructor combines mastery-based approaches with an ungrading scheme.

Prerequisites: MATH 250; CSCI 134 or equivalent prior experience with computer programming (in any language). These prerequisites will be strictly enforced.

Enrollment Limit: 24
Enrollment Preferences: Preference given to students who need to make use of linear algebra in their major fields of study. First-day attendance is required.

Expected Class Size: 24
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This course involves developing the formal mathematical language of linear algebra. It also involves using quantitative tools to solve problems relating to a wide range of applications in the physical and social sciences.

Not offered current academic year
Ordinary differential equations (ODEs) frequently arise as models of phenomena in the natural and social sciences. This course presents core ideas of ODEs from an applied standpoint. Topics covered early in the course may include numerical solutions, separation of variables, integrating factors, and constant coefficient linear equations. Later, we will focus on nonlinear ODEs, for which it is usually impossible to find analytical solutions. Tools from dynamical systems will be introduced to allow us to obtain information about the behavior of the ODEs without explicitly knowing the solution.

Requirements/Evaluation: quizzes/exams, problem sets, participation

Prerequisites: MATH 150/151 and MATH 250

Enrollment Limit: 40

Enrollment Preferences: discretion of the instructor

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: 300-level mathematics course

Fall 2024
LEC Section: 01 TR 9:55 am - 11:10 am Julie C. Blackwood

Spring 2025
LEC Section: 01 TR 8:30 am - 9:45 am Bhagya Athukorallage

Applied mathematics is an expansive field that uses mathematical methods to explore problems that arise in biology, physics, engineering, and many other disciplines. In this course, we will explore a diversity of methods that may include stochastic processes, optimization, signal processing, and numerical analysis. We will also explore how these methods can be utilized to understand questions in other disciplines.

Requirements/Evaluation: This course will have some combination of problem sets, presentations, exams, and a final project

Prerequisites: Differential equations (Math 309) or permission of the instructor

Enrollment Limit: 10

Enrollment Preferences: If over-enrolled, the instructor will request a statement of interest

Expected Class Size: 10

Grading: yes pass/fail option, no fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: Mathematics course

Not offered current academic year

The study of numbers dates back thousands of years, and is fundamental in mathematics. In this course, we will investigate both classical and modern questions about numbers. In particular, we will explore the integers, and examine issues involving primes, divisibility, and congruences. We will also look at the ideas of number and prime in more general settings, and consider fascinating questions that are simple to understand, but can be quite difficult to answer. This course will include a significant focus on mathematical proof writing and problem solving skills. This includes writing clear and rigorous mathematical proofs, clearly explaining mathematical ideas verbally and in writing, determining how to approach certain types of problems, looking for patterns and making conjectures, and asking good questions about the implications of certain ideas and theorems.

Requirements/Evaluation: Problem sets, project, and exams

Prerequisites: MATH 250 or permission of instructor

Enrollment Limit: 19

Enrollment Preferences: If course is over-enrolled, enrollment preference will be based on answers to a questionnaire. Some preference will be given to students who have not yet had Math 355.

Expected Class Size: 19
Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (WS) (QFR)

Writing Skills Notes: Students will complete weekly problem sets, with a strong emphasis on proof writing, as well as 2 5-10 page papers/projects. There will be feedback given on mathematical writing as well as accuracy, and discussion time during class on writing in math.

Quantitative/Formal Reasoning Notes: This course requires working with various number systems, performing explicit computations, and proving mathematical results using logical reasoning practices.

Spring 2025
LEC Section: 01 TR 9:55 am - 11:10 am Allison Pacelli

MATH 314 (F) Cryptography (QFR)
We will discuss some classical ciphers, current asymmetric cryptosystems (DES, AES, Rijndael), public key cryptosystems (RSA, Diffie-Hellman key exchange, ElGamal), and Error Correcting Codes. We will devote a substantial part of the semester covering the necessary mathematical background from number theory and asymptotic analysis. Time permitting, we may also discuss some special topics, such as primality testing (including the polynomial-time AKS algorithm), quantum computers, hash functions, digital signatures, zero-knowledge proofs, information theory, and elliptic curve cryptography.

Requirements/Evaluation: exams, problem sets, quizzes
Prerequisites: MATH 250 or permission of instructor.
Enrollment Limit: 30
Enrollment Preferences: Juniors and seniors.
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: The course will contain mathematical proofs.

Not offered current academic year

MATH 317 (F) Introduction to Operations Research (QFR)
In the first N math classes of your career, you can be misled as to what the world is truly like. How? You're given exact problems and told to find exact solutions. The real world is sadly far more complicated. Frequently we cannot exactly solve problems; moreover, the problems we try to solve are themselves merely approximations to the world! We are forced to develop techniques to approximate not just solutions, but even the statement of the problem. Additionally, we often need the solutions quickly. Operations Research, which was born as a discipline during the tumultuous events of World War II, deals with efficiently finding optimal solutions. In this course we build analytic and programming techniques to efficiently tackle many problems. We will review many algorithms from earlier in your mathematical or CS career, with special attention now given to analyzing their run-time and seeing how they can be improved. The culmination of the course is a development of linear programming and an exploration of what it can do and what are its limitations. For those wishing to take this as a Stats course, the final project must have a substantial stats component approved by the instructor.
Prerequisites: Linear Algebra (MATH 250) and one other 200-level or higher CSCI, MATH or STATS course, or permission of the instructor.
Requirements/Evaluation: homework, exams, projects
Prerequisites: MATH 150, MATH 250 and one other 200-level or higher CSCI, MATH or STATS course, or permission from the instructor.
Enrollment Limit: 40
Enrollment Preferences: Computer Science, Mathematics and Statistics majors
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Unit Notes: http://web.williams.edu/Mathematics/sjmiller/public_html/317/
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: 300 level mathematics course.

Not offered current academic year
MATH 319 (S) Integrative Bioinformatics, Genomics, and Proteomics Lab (QFR)

Cross-listings: BIOL 319 / CHEM 319 / CSCI 319 / PHYS 319

Secondary Cross-listing
What can computational biology teach us about cancer? In this lab-intensive experience for the Genomics, Proteomics, and Bioinformatics program, computational analysis and wet-lab investigations will inform each other, as students majoring in biology, chemistry, computer science, mathematics/statistics, and physics contribute their own expertise to explore how ever-growing gene and protein data-sets can provide key insights into human disease. In this course, we will take advantage of one well-studied system, the highly conserved Ras-related family of proteins, which play a central role in numerous fundamental processes within the cell. The course will integrate bioinformatics and molecular biology, using database searching, alignments and pattern matching, and phylogenetics to reconstruct the evolution of gene families by focusing on the gene duplication and gene rearrangements that have occurred over the course of eukaryotic speciation. By utilizing high throughput approaches to investigate genes involved in the inflammatory and MAPK signal transduction pathways in human colon cancer cell lines, students will uncover regulatory mechanisms that are aberrantly altered by siRNA knockdown of putative regulatory components. This functional genomic strategy will be coupled with independent projects using phosphorylation-state specific antisera to test our hypotheses. Proteomic analysis will introduce the students to de novo structural prediction and threading algorithms, as well as data-mining approaches and Bayesian modeling of protein network dynamics in single cells. Flow cytometry and mass spectrometry may also be used to study networks of interacting proteins in colon tumor cells.

Class Format: The lab for this course will meet for two afternoons per week. Some lab sessions will be shorter than 3 hours.

Requirements/Evaluation: lab participation, several short homework assignments, one lab report, a programming project, and a grant proposal

Prerequisites: BIOL 202; students who have not taken BIOL 202 but have taken BIOL 101 and a CSCI course, or CSCI/PHYS 315, may enroll with permission of instructor. No prior computer programming experience is required.

Enrollment Limit: 12

Enrollment Preferences: seniors, then juniors, then sophomores

Expected Class Size: 12

Grading: yes pass/fail option, no fifth course option

Distributions: (D3) (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:
BIOL 319(D3) CHEM 319(D3) CSCI 319(D3) PHYS 319(D3) MATH 319(D3)

Quantitative/Formal Reasoning Notes: Through lab work, homework sets and a major project, students will learn or further develop their skills in programming in Python, and about the basis of Bayesian approaches to phylogenetic tree estimation.

Attributes: BIGP Courses BIMO Interdepartmental Electives

Not offered current academic year

MATH 321 (S) Knot Theory (QFR)

Take a piece of string, tie a knot in it, and glue the ends together. The result is a knotted circle, known as a knot. For the last 100 years, mathematicians have studied knots, asking such questions as, ”Given a nasty tangled knot, how do you tell if it can be untangled without cutting it open?” Some of the most interesting advances in knot theory have occurred in the last ten years. This course is an introduction to the theory of knots. Among other topics, we will cover methods of knot tabulation, surfaces applied to knots, polynomials associated to knots, and relationships between knot theory and chemistry and physics. In addition to learning the theory, we will look at open problems in the field.

Requirements/Evaluation: problem sets, midterms, a paper and a final exam

Prerequisites: MATH 250 or permission of instructor

Enrollment Limit: 30

Enrollment Preferences: seniors, junior, sophomores, first year

Expected Class Size: 25

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is a quantitative course.

Not offered current academic year
MATH 326 (F) Differential Geometry (QFR)
Differential Geometry is the study of curvature. In turn, curvature is the heart of geometry. The goal of this course is to start the study of curvature, concentrating on the curvature of curves and of surfaces, leading to the deep Gauss-Bonnet Theorem, which links curvature with topology.

Class Format: lecture
Requirements/Evaluation: Evaluation will be based primarily on problem sets, midterms and a final exam
Prerequisites: MATH 250
Enrollment Limit: 30
Enrollment Preferences: Preference to mathematics majors.
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: It is a mathematics upper level course.

Fall 2024
LEC Section: 01 MWF 11:00 am - 11:50 am Ivo Terek

MATH 328 (S) Combinatorics (QFR)
Combinatorics is a branch of mathematics that focuses on enumerating, examining, and investigating the existence of discrete mathematical structures with certain properties. This course provides an introduction to the fundamental structures and techniques in combinatorics including enumerative methods, generating functions, partition theory, the principle of inclusion and exclusion, and partially ordered sets.

Class Format: interactive activities and discussion
Requirements/Evaluation: quizzes/exams, homework, activities
Prerequisites: "MATH 250, and MATH 200 or permission of instructor"
Enrollment Limit: 30
Enrollment Preferences: discretion of the instructor
Expected Class Size: 25
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This is an upper level course in mathematics

Not offered current academic year

MATH 329 (S) Discrete Geometry (QFR)
Discrete geometry is one of the oldest and most consistently vibrant areas of mathematics, stretching from the Platonic Solids of the ancient Greeks to the modern day applications of convex optimization and linear programming. In this tutorial we will learn about polygons and their higher-dimensional cousins, polyhedra and polytopes, and the various ways to describe, compute, and classify such objects. We will learn how these objects and ideas can be applied to other areas, from computation and optimization to studying areas of math like algebraic geometry. Throughout this course we will be engaging with mathematical work and literature from as old as 500 BCE and as recent as "posted to the internet yesterday."

Requirements/Evaluation: Evaluation will be based primarily on participation, problem sets, oral presentations, a written midterm exam, an oral final exam, and a final project
Prerequisites: MATH 200 or Math 250, or permission of instructor
Enrollment Limit: 10
Enrollment Preferences: First-years and sophomores
Expected Class Size: 10
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: All of the content in this course is quantitative or formal reasoning.

MATH 331  (F)  The little Questions  (QFR)
Using math competitions such as the Putnam Exam as a springboard, in this class we follow the dictum of the Ross Program and "think deeply of simple things". The two main goals of this course are to prepare students for competitive math competitions, and to get a sense of the mathematical landscape encompassing elementary number theory, combinatorics, graph theory, and group theory (among others). While elementary frequently is not synonymous with easy, we will see many beautiful proofs and "a-ha" moments in the course of our investigations. Students will be encouraged to explore these topics at levels compatible with their backgrounds.

Requirements/Evaluation: homework, classwork, and exams
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 30
Enrollment Preferences: members or alternates of the Putnam team, Mathematics, Physics or Computer Science majors
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Unit Notes: http://web.williams.edu/Mathematics/sjmiller/public_html/331Fa24/
Distributions: (D3)  (QFR)
Quantitative/Formal Reasoning Notes: This is an upper level math course where students learn advanced material and solve challenging problems.

Fall 2024
LEC Section: 01    MWF 10:00 am - 10:50 am     Steven J. Miller

MATH 332  (F)  Topics in Applied Linear Algebra  (QFR)
This course focuses on applications of Linear Algebra. We will start with a review of the material covered in Math 250, then move on to more advanced topics and applications. We will cover Singular Value Decomposition (SVD), QR factorization, Cholesky factorization, Least Squares problems, the Taylor approximation, the Regression model, Clustering techniques, as well as Linear Dynamical Systems and some of their applications.

Requirements/Evaluation: Homework assignments and exams.
Prerequisites: Math 250
Enrollment Limit: 30
Enrollment Preferences: Mathematics Majors, Seniors
Expected Class Size: 15
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)  (QFR)
Quantitative/Formal Reasoning Notes: This is an advanced mathematics course, building upon the core course Math 250 - Linear Algebra.

Not offered current academic year

MATH 334  (S)  Graph Theory  (QFR)
A graph is a collection of vertices, joined together by edges. In this course, we will study the sorts of structures that can be encoded in graphs, along with the properties of those graphs. We’ll learn about such classes of graphs as multi-partite, planar, and perfect graphs, and will see applications to such optimization problems as minimum colorings of graphs, maximum matchings in graphs, and network flows.

Requirements/Evaluation: problem sets, exams, and a short final project
Prerequisites: MATH 200 or MATH 250
Enrollment Limit: 30
Enrollment Preferences: Math majors
Expected Class Size: 25
**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This course involves the writing of mathematical proofs.

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**Spring 2025**

LEC Section: 01    MWF 9:00 am - 9:50 am     Ralph E. Morrison

**MATH 338 (F) Intermediate Logic** (QFR)

**Cross-listings:** PHIL 338

**Secondary Cross-listing**

In this course, we will begin with an in-depth study of the theory of first-order logic. We will first get clear on the formal semantics of first-order logic and various ways of thinking about formal proof: natural deduction systems, semantic tableaux, axiomatic systems and sequent calculi. Our main goal will be to prove things about this logical system rather than to use this system to think about ordinary language arguments. In this way the goal of the course is significantly different from that of Logic and Language (PHIL 203). Students who have taken PHIL 203 will have a good background for this class, but students who are generally comfortable with formal systems need not have taken PHIL 203. We will prove soundness and completeness, compactness, the Lowenheim-Skolem theorems, undecidability and other important results about first-order logic. As we go through these results, we will think about the philosophical implications of first-order logic. From there, we will look at extensions of and/or alternatives to first-order logic. Possible additional topics would include: modal logic, the theory of counterfactuals, alternative representations of conditionals, the use of logic in the foundations of arithmetic and Godel's Incompleteness theorems. Student interest will be taken into consideration in deciding what additional topics to cover.

**Requirements/Evaluation:** problem sets and exams

**Prerequisites:** some class in which student has studied formal reasoning

**Enrollment Limit:** 20

**Enrollment Preferences:** Philosophy majors; juniors and seniors

**Expected Class Size:** 15

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:

PHIL 338(D2) MATH 338(D3)

**Quantitative/Formal Reasoning Notes:** This is a class in Formal Logic. PHIL 203 satisfies the QFR requirement. If anything, this class will be significantly more formal.

**Attributes:** PHIL Contemp Metaphysics + Epistemology Courses

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**Fall 2024**

SEM Section: 01    TF 2:35 pm - 3:50 pm     Keith E. McPartland

**MATH 340 Applications of Mathematics to the Real World** (QFR)

Often for real world applications one does not need to find the optimal solution, which can be extremely difficult, but instead just find something close, or at least better than what is currently being done. We will develop material and techniques from mathematics, statistics and allied fields with an eye to applications. In addition to standard homework assignments and exams there will be a group project where students will work with a local business, write a report and present the results. Pre-requisites are multivariable calculus and linear algebra, or permission of the instructor. Knowledge of some statistics or programming is beneficial but not required.

**Class Format:** In addition to standard lectures and assignments, we will be partnering with local businesses to apply mathematics to solve real world problems.

**Requirements/Evaluation:** Lectures and class participation, homework, exams and encouragement to do a project. We already have several local businesses with projects for students. Working on a project will entail meeting with officials from the company, clearly defining what the problem is, and writing a solution. This will include a presentation, a write-up, and potentially implementable code. Based on previous similar courses, these papers
The historical roots of probability lie in the study of games of chance. Modern probability, however, is a mathematical discipline that has wide applications in a myriad of other mathematical and physical sciences. Drawing on classical gaming examples for motivation, this course will present axiomatic and mathematical aspects of probability. Included will be discussions of random variables (both discrete and continuous), distribution and expectation, independence, laws of large numbers, and the well-known Central Limit Theorem. Many interesting and important applications will also be presented, including some from classical Poisson processes, random walks and Markov Chains.

Requirements/Evaluation: homework, classwork, and exams
Prerequisites: MATH 150 and MATH 250 or permission of the instructor
Enrollment Limit: 50
Enrollment Preferences: Priority will be given to Mathematics majors and to Statistics Majors.
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This is a 300-level Math/Stat course.

Fall 2024
LEC Section: 01  MWF 12:00 pm - 12:50 pm  Mihai Stoiciu

Spring 2025
LEC Section: 01  TR 9:55 am - 11:10 am  Steven J. Miller

MATH 342  (F) Logic  (QFR)
This course will introduce the main ideas and basic results of mathematical logic, and explain their applications to other areas of mathematics and computer science. We will begin with a study of first-order logic, covering structures and definability, theories, models and categoricity, as well as formal proofs. We will prove Gödel's completeness and compactness theorems and the Lowenheim-Skolem theorems. The course will briefly dive into computability theory, enough to prove Gödel's Incompleteness theorems and basic undecidability results.

Requirements/Evaluation: Evaluation based on homework, exams, and class participation.
Prerequisites: Math 250 - Linear Algebra
Enrollment Limit: 20
Enrollment Preferences: Junior and Senior Math Majors
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: Mathematics course in logic and applications.

Not offered current academic year

MATH 344 (S) The Mathematics of Sports (QFR)
The purpose of this class is to use sports as a springboard to study applications of mathematics, especially in gathering data to build and test models and develop predictive statistics. Examples will be drawn from baseball, basketball, cross country, football, hockey, soccer, track, as well as class choices. Pre-requisites are linear algebra (Math 250) and either a 200 level statistics class or a 100 level programming class, or permission of the instructor.

Requirements/Evaluation: Homework, exams, projects
Prerequisites: Math 250: Linear Algebra
Enrollment Limit: 40
Enrollment Preferences: None. If the course is over-enrolled preference will be given to math and stats majors, and then if needed by performance on a small assignment.
Expected Class Size: 30
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is a 300 level mathematics course.

Not offered current academic year

MATH 345 (S) Introduction to Numerical Analysis (QFR)
Numerical analysis is the study of algorithms that use numerical approximation to solve problems which arise in scientific applications. This course provides an introduction to the theory, development, and analysis of algorithms for obtaining numerical solutions. We will also use mathematical software to facilitate numerical experimentation. Topics discussed in the course include: Error Analysis and Convergence Rates of Algorithms; Root Finding for Nonlinear Equations; Approximating Functions; Numerical Differentiation and Integration; Numerical Solutions of Ordinary Differential Equations; Iterative Methods for Solving Linear Systems.

Requirements/Evaluation: Evaluation will be based on homework, projects, and exams.
Prerequisites: Math 250 and Math 150/151
Enrollment Limit: 30
Enrollment Preferences: Mathematics and Statistics majors.
Expected Class Size: 15
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is an advanced mathematics class that will cover the fundamental ideas of Numerical Analysis. The students will study in depths various algorithms that provide numerical solutions to various questions in science.

Spring 2025
LEC Section: 01 MWF 11:00 am - 11:50 am Christina Athanasouli

MATH 349 (F) Operations of Order (WS) (QFR)
One of the greatest challenges in mathematics is justifying interchanging orders of operations. Most of the time you cannot switch orders. Frequently this is obvious: the square root of a sum is typically not the sum of the square roots; however, there are many important situations where orders can be reversed. The purpose of this class is to highlight some of the difficulties and dangers in such attempts. This will be a writing intensive course, where we work on content for a book that collects counter-examples and theorems in one convenient place while also showcasing the utility of switching orders. We will discuss at great lengths how to do engaging, technical writing, keeping in mind the content and the audience. Students will receive feedback from the professor and probably other professional mathematicians and editors.
**Requirements/Evaluation:** Mix of homework, exams, and writing, including at least one chapter (consisting of theory, examples, images, homework problem creation and solutions).

**Prerequisites:** Math 250 or permission of the instructor.

**Enrollment Limit:** 19

**Enrollment Preferences:** If over-enrolled, students will be chosen uniformly at random.

**Expected Class Size:** 10

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (WS) (QFR)

**Writing Skills Notes:** Students will be working closely with me and colleagues, receiving feedback on their writing from numerous sources (myself, editors, experts in the field), and their work will be part of the final, published manuscript. We will have numerous discussions about how to write, taking into account the audience and the content.

**Quantitative/Formal Reasoning Notes:** This is a 300 level math course.

Not offered current academic year

**MATH 350  (F)(S)  Real Analysis  (QFR)**

Why is the product of two negative numbers positive? Why do we depict the real numbers as a line? Why is this line continuous, and what do we mean when we say that? Perhaps most fundamentally, what is a real number? Real analysis addresses such questions, delving into the structure of real numbers and functions of them. Along the way we’ll discuss sequences and limits, series, completeness, compactness, derivatives and integrals, and metric spaces. Results covered will include the Cantor-Schroeder-Bernstein theorem, the monotone convergence theorem, the Bolzano-Weierstrass theorem, the Cauchy criterion, Dirichlet’s and Riemann’s rearrangement theorem, the Heine-Borel theorem, the intermediate value theorem, and many others. This course is excellent preparation for graduate studies in mathematics, statistics, and economics.

**Requirements/Evaluation:** Problem sets and exams.

**Prerequisites:** MATH 250 or permission of instructor.

**Enrollment Limit:** 40

**Enrollment Preferences:** Juniors and Seniors.

**Expected Class Size:** 25

**Grading:** no pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This is an advanced mathematics course.

Fall 2024
LEC Section: 01  MR 2:35 pm - 3:50 pm  Leo Goldmakher

Spring 2025
LEC Section: 01  TR 11:20 am - 12:35 pm  Cesar E. Silva

**MATH 351  (S)  Applied Real Analysis  (QFR)**

This course is designed to introduce students to the underpinnings of real analysis, primarily in the context of Fourier series. By the end of the semester people will be comfortable making epsilon and delta type arguments. These types of arguments are one of the main pillars of modern mathematics. In a similar way, Fourier series and their generalizations are one of the pillars of the modern digital world.

**Requirements/Evaluation:** homework, classwork, and exams

**Prerequisites:** MATH 150 and MATH 250 or permission of the instructor.

**Enrollment Limit:** 50

**Enrollment Preferences:** Seniors

**Expected Class Size:** 20

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)
MATH 355 (F)(S) Abstract Algebra (QFR)
Algebra gives us tools to solve equations. The integers, the rationals, and the real numbers have special properties which make algebra work according to the circumstances. In this course, we generalize algebraic processes and the sets upon which they operate in order to better understand, theoretically, when equations can and cannot be solved. We define and study abstract algebraic structures such as groups, rings, and fields, as well as the concepts of factor group, quotient ring, homomorphism, isomorphism, and various types of field extensions. This course introduces students to abstract rigorous mathematics.

Requirements/Evaluation: Problem sets and exams
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 30
Enrollment Preferences: Students who have officially declared a major that requires Math 355.
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: 300-level math course
MATH 374 (F) Topology (QFR)

In Real Analysis you learned about metric spaces---any set of objects endowed with a way of measuring distance---and the topology of sets in such spaces (open, closed, bounded, etc). In this course we flip this on its head: we explore how to develop analysis (limits, continuity, etc) in spaces where the topology is known but the metric is not. This will lead us to a bizarre and fascinating version of geometry in which we cannot distinguish between shapes that can be continuously deformed into one another. Not only does this theory turn out to be beautiful in the abstract, it plays an important role in math, physics, and data analysis. This course is excellent preparation for graduate programs in mathematics.

Requirements/Evaluation: Problem sets, exams, an expository essay.
Prerequisites: MATH 350 or 351; not open to students who have taken MATH 323. If you didn't cover metric spaces in real analysis, that's OK!
Enrollment Limit: 30
Enrollment Preferences: Juniors and seniors
Expected Class Size: 20
Grading: no pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: It's math.

Fall 2024
LEC Section: 01 TR 8:30 am - 9:45 am Shikha Singh
LEC Section: 02 TR 9:55 am - 11:10 am Shikha Singh

MATH 382 (S) Fourier Analysis (QFR)

Fourier analysis is the study of waves and frequencies. More precisely, the goal of Fourier analysis is to decompose a complicated function into a simple combination of pure waves, thereby gleaning insight into the behavior of the function itself. It's difficult to overstate the impact of this branch of mathematics; it is foundational throughout theoretical mathematics (e.g., to study the distribution of prime numbers), applied mathematics (e.g., to solve differential equations), physics (e.g., to study properties of light and sound), computer science (e.g., to compute with large integers and matrices), audio engineering (e.g., to pitch-correcting algorithms), medical science (e.g., throughout radiology), etc. The goal of this course is to cover the basic theory (fourier series, the fourier transform, the fast fourier transform) and explore a number of applications, including Dirichlet's theorem on primes in arithmetic progressions, the isoperimetric inequality, the heat equation, and Heisenberg's uncertainty principle.

Class Format: Every week, each student will either give a lecture (based on provided readings) or explain solutions to selected problems.
Requirements/Evaluation: Evaluation will be based on lectures and presentation of problem solutions.
Prerequisites: MATH 350 or MATH 351 or permission of instructor.
Enrollment Limit: 10
Enrollment Preferences: By lottery.
Expected Class Size: 10
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: It's math!

Fall 2024
LEC Section: 01 TR 11:20 am - 12:35 pm Leo Goldmakher

Spring 2025
TUT Section: T1 TBA Leo Goldmakher
MATH 383 (F) Complex Analysis  (QFR)
The calculus of complex-valued functions turns out to have unexpected simplicity and power. As an example of simplicity, every complex-differentiable function is automatically infinitely differentiable. As examples of power, the so-called "residue calculus" permits the computation of "impossible" integrals, and "conformal mapping" reduces physical problems on very general domains to problems on the round disc. The easiest proof of the Fundamental Theorem of Algebra, not to mention the first proof of the Prime Number Theorem, used complex analysis.

Requirements/Evaluation: homework, classwork, and exams
Prerequisites: MATH 350 or MATH 351 or permission of instructor
Enrollment Limit:  40
Enrollment Preferences:  40
Expected Class Size:  30
Grading:  yes pass/fail option,   yes fifth course option
Unit Notes: this course is not a senior seminar, so it does not fulfill the senior seminar requirement for the Math major
Distributions:  (D3)  (QFR)
Quantitative/Formal Reasoning Notes: Advanced mathematics course with weekly or daily problem sets.
Not offered current academic year

MATH 389  (S)  Advanced Analysis  (WS) (QFR)
This course further develops and explores topics and concepts from real analysis, with special emphasis on introducing students to subject matter and techniques that are useful for graduate study in mathematics or an allied field. Material will be drawn, based on student interest, from many areas, including analytic number theory, Fourier series and harmonic analysis, generating functions, differential equations and special functions, integral operators, equidistribution theory and probability, random matrix theory and probabilistic methods. This will be an intense, fast paced class which will give a flavor for graduate school. In addition to standard homework problems, students will also write reviews for MathSciNet, referee papers for journals, write programs in SAGE or Mathematica to investigate and conjecture, and read classic and current research papers.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on homework, classwork, and exams
Prerequisites: MATH 350 or 351 and one additional 300-level MATH course, or permission of instructor
Enrollment Limit:  30
Enrollment Preferences:  students interested in graduate school in mathematics or an allied discipline
Expected Class Size:  15
Grading:  yes pass/fail option,   yes fifth course option
Unit Notes: http://web.williams.edu/Mathematics/sjmiller/public_html/389/
Distributions:  (D3)  (WS) (QFR)
Writing Skills Notes: Students will improve and expand their writing skills by taking course material as a starting point and writing chapters for a book under contract with the American Mathematical Society. This will involve numerous iterations of the content, with feedback both from the professor and from an editor.
Quantitative/Formal Reasoning Notes: This is a post-core 300 level math class.

Spring 2025
LEC Section: 01    TR 11:20 am - 12:35 pm     Steven J. Miller

MATH 390  Undergraduate Research Topics in Algebra  (QFR)
The well-known trace map on matrices can be generalized to a map on other algebraic objects. Undergraduates, graduates students and experts in Representation Theory, Commutative Algebra and Algebraic Geometry have been driving recent developments in the theory of trace modules and finding exciting new applications in all of these these fields. This course will serve as an introduction to mathematical research with the aim of producing original research in modern trace theory. Students in this tutorial will read and synthesize research papers, discuss the formation of research questions in pure mathematics, and engage in original mathematical research.
Requirements/Evaluation: oral presentations; writing assignments (summarizing papers, reflections on mathematical research, original research); participation in the course project

Prerequisites: Math 355

Enrollment Limit: 10

Enrollment Preferences: Juniors and Seniors

Expected Class Size: 7

Grading:

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is post-core math class; students will be required to produce mathematical proofs.

Not offered current academic year

MATH 393 (S) Research Topics in Combinatorics (WS) (QFR)

Combinatorics provides techniques and tools to enumerate, examine, and investigate the existence of discrete mathematical structures with certain properties. There are numerous areas of applications including algebra, discrete geometry, and number theory. In this project-based research course students will work in small groups to learn combinatorial techniques and tools in order to develop research questions and begin tackling unsolved problems in combinatorics.

Requirements/Evaluation: Students will be evaluated through written drafts of a manuscript and its revisions and multiple in-class presentation.

Prerequisites: Math 355

Enrollment Limit: 19

Enrollment Preferences: Post-core mathematics majors

Expected Class Size: 19

Grading: no pass/fail option, no fifth course option

Distributions: (D3) (WS) (QFR)

Writing Skills Notes: The main goal of this course is to undertake original research in combinatorics, as such student assessment is based on developing positive collaboration skills, and improving technical written and oral skills in mathematics through manuscript draft submissions and in-class presentations. Students will provide multiple drafts of their manuscript and in right of this the course will be writing intensive.

Quantitative/Formal Reasoning Notes: The main goal of this course is to undertake original research in the math field of mathematics. See above for more details.

Not offered current academic year

MATH 394 (S) Galois Theory (QFR)

Some equations--such as x^5-1=0--are easy to solve. Others--such as x^5-x-1=0--are very hard, if not impossible (using finite combinations of standard mathematical operations). Galois discovered a deep connection between field theory and group theory that led to a criterion for checking whether or not a given polynomial can be easily solved. His discovery also led to many other breakthroughs, for example proving the impossibility of squaring the circle or trisecting a typical angle using compass and straightedge. From these not-so-humble beginnings, Galois theory has become a fundamental concept in modern mathematics, from topology to number theory. In this course we will develop the theory and explore its applications to other areas of math.

Requirements/Evaluation: problem sets and oral exams

Prerequisites: MATH 355

Enrollment Limit: 30

Enrollment Preferences: Juniors and seniors

Expected Class Size: 15

Grading: no pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is a math class

Not offered current academic year
MATH 397 (F) Independent Study: Mathematics
Directed 300-level independent study in Mathematics.
Prerequisites: permission of department
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)
Not offered current academic year

MATH 398 (S) Independent Study: Mathematics
Directed 300-level independent study in Mathematics.
Prerequisites: permission of department
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)
Not offered current academic year

MATH 403 (F) Measure and Ergodic Theory (QFR)
An introduction to measure theory and ergodic theory. Measure theory is a generalization of the notion of length and area, and has been used in the study of stochastic (probabilistic) systems. The course covers the construction of Lebesgue and Borel measures, measurable functions, and Lebesgue integration. Ergodic theory studies the probabilistic behavior of dynamical systems as they evolve through time, and is based on measure theory. The course will cover basic notions, such as ergodic transformations, weak mixing, mixing, Bernoulli transformations, and transformations admitting and not admitting an invariant measure. There will be an emphasis on specific examples such as group rotations, the binary odometer transformations, and rank-one constructions. The Ergodic Theorem will also be covered, and will be used to illustrate notions and theorems from measure theory.
Requirements/Evaluation: Homework and exams
Prerequisites: MATH 350 or MATH 351 or permission of instructor
Enrollment Limit: 25
Enrollment Preferences: Mathematics majors
Expected Class Size: 15-20
Grading: yes pass/fail option, yes fifth course option
Unit Notes: senior major course
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: It is an advanced mathematics course.

Fall 2024
LEC Section: 01 MR 2:35 pm - 3:50 pm Cesar E. Silva

MATH 407 (F) Dance of the Primes (QFR)
Prime numbers are the building blocks for all numbers and hence for most of mathematics. Though there are an infinite number of them, how they are spread out among the integers is still quite a mystery. Even more mysterious and surprising is that the current tools for investigating prime numbers involve the study of infinite series. Function theory tells us about the primes. We will be studying one of the most amazing functions known: the Riemann Zeta Function. Finding where this function is equal to zero is the Riemann Hypothesis and is one of the great, if not greatest, open problems in mathematics. Somehow where these zeros occur is linked to the distribution of primes. We will be concerned with why anyone would care about this conjecture. More crassly, why should solving the Riemann Hypothesis be worth one million dollars? (Which is what you will get if you solve it, beyond the eternal fame and glory.)
Requirements/Evaluation: exams and weekly homework assignments
Prerequisites: MATH 350 or MATH 351, and MATH 355
Enrollment Limit: 30
Enrollment Preferences: seniors
Expected Class Size: 10
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: It is a math course.
Not offered current academic year

MATH 408 (F) L-Functions and Sphere Packing  (WS) (QFR)
Optimal packing problems arise in many important problems, and have been a source of excellent mathematics for centuries. The Kepler Problem (what is the most efficient way to pack balls in three-space) is a good example. The original formulation has been used in such diverse areas as stacking cannonballs on battleships to grocers preparing fruit displays, and its generalizations allow the creation of powerful error detection and correction codes. While the solution of the Kepler Problem is now known, the higher dimensional version is very much open. There has been remarkable progress in the last few years, with number theory playing a key role in these results. We will develop sufficient background material to understand many of these problems and the current state of the field. Pre-requisites are real analysis.

Requirements/Evaluation: Class participation, homework, exams and participation in writing a textbook on the material. Each student will be responsible for working on a chapter of a book based on this material. In addition to obtaining critical writing feedback from myself and my co-author (who is a world expert in the subject), depending on timing we will also be able to share comments from an editor of a major publishing house or a referee. Chapters can range from short snapshots of a subject, on the order of 5 pages, to longer technical derivations of perhaps 10-30 pages.

Prerequisites: Math 350 or 351
Enrollment Limit: 40
Enrollment Preferences: Senior math majors, students planning on graduate study in a STEM field
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (WS) (QFR)

Writing Skills Notes: Students will improve and expand their writing skills by taking course material as a starting point and writing chapters for a book under contract with the American Mathematical Society. This will involve numerous iterations of the content, with feedback both from the professor and from an editor.

Quantitative/Formal Reasoning Notes: This is a 400 level math class

Fall 2024
LEC Section: 01  MWF 11:00 am - 11:50 am  Steven J. Miller

MATH 409 (F) The Little Questions  (QFR)
Using math competitions such as the Putnam Exam as a springboard, in this class we follow the dictum of the Ross Program and "think deeply of simple things". The two main goals of this course are to prepare students for competitive math competitions, and to get a sense of the mathematical landscape encompassing elementary number theory, combinatorics, graph theory, and group theory (among others). While elementary frequently is not synonymous with easy, we will see many beautiful proofs and 'a-ha' moments in the course of our investigations. Students will be encouraged to explore these topics at levels compatible with their backgrounds.

Requirements/Evaluation: Homework, exams, presentations.
Prerequisites: Real Analysis (either Math 350 or 351) and Abstract Algebra (Math 355), or permission of the instructor.
Enrollment Limit: 30
Enrollment Preferences: Math/stat senior majors
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This is a 400 level math class.
MATH 411  (S)  Commutative Algebra  (QFR)

Commutative Algebra is an essential area of mathematics that provides indispensable tools to many areas, including Number Theory and Algebraic Geometry. This course will introduce you to the fundamental concepts for the study of commutative rings, with a special focus on the notion of "prime ideals," and how they generalize the well-known notion of primality in the set of integers. Commutative algebra has applications ranging from algebraic geometry to coding theory. For example, one can use commutative algebra to create error correcting codes. It is perhaps most often used, however, to study curves and surfaces in different spaces. To understand these structures, one must study polynomial rings over fields. This course will be an introduction to commutative algebra. Possible topics include polynomial rings, localizations, primary decomposition, completions, and modules.

Requirements/Evaluation:  homework and exams
Prerequisites:  MATH 355 or permission of instructor
Enrollment Limit:  25
Enrollment Preferences:  Math majors
Expected Class Size:  15
Grading:  yes pass/fail option,  yes fifth course option
Distributions:  (D3)  (QFR)
Quantitative/Formal Reasoning Notes:  It is a 400-level math course

Spring 2025
LEC Section: 01    MWF 9:00 am - 9:50 am     Susan R. Loepp

MATH 412  (S)  Mathematical Biology  (QFR)

Cross-listings:  BIOL 420
Primary Cross-listing

This course will provide an introduction to the many ways in which mathematics can be used to understand, analyze, and predict biological dynamics. We will learn how to construct mathematical models that capture essential properties of biological processes while maintaining analytic tractability. Analytic techniques, such as stability and bifurcation analysis, will be introduced in the context of both continuous and discrete time models. Additionally, students will couple these analytic tools with numerical simulation to gain a more global picture of the biological dynamics. Possible biological applications may include, but are not limited to, single and multi-species population dynamics, neural and biological oscillators, tumor cell growth, and infectious disease dynamics.

Requirements/Evaluation:  problem sets, quizzes/exams, participation, final project and paper
Prerequisites:  MATH 250 and MATH 309, or permission of instructor
Enrollment Limit:  30
Enrollment Preferences:  preference for senior math/stats major and also based on an interest statement
Expected Class Size:  30
Grading:  yes pass/fail option,  no fifth course option
Distributions:  (D3)  (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:
MATH 412(D3)  BIOL 420(D3)
Quantitative/Formal Reasoning Notes:  The course will introduce methods for developing and analyzing mathematical models.

Spring 2025
LEC Section: 01    TR 9:55 am - 11:10 am     Julie C. Blackwood

MATH 413  (S)  Computational Algebraic Geometry  (QFR)

Algebraic geometry is the study of shapes described by polynomial equations. It has been a major part of mathematics for at least the past two
hundred years, and has influenced a tremendous amount of modern mathematics, ranging from number theory to robotics. In this course, we will develop the Ideal-Variety Correspondence that ties geometric shapes to abstract algebra, and will use computational tools to explore this theory in a very explicit way.

**Requirements/Evaluation:** Evaluation will be based on weekly problem sets, three exams, and final project. Any students who have taken Math 411 should consult with the instructor before enrolling in this course.

**Prerequisites:** Math 355

**Enrollment Limit:** 30

**Enrollment Preferences:** Preference given to senior math majors

**Expected Class Size:** 20

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This course is all quantitative and formal reasoning.

Not offered current academic year

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**MATH 415 (F) Advanced Matrix Analysis** (QFR)

This course will start with a review of various attributes of matrices (determinants, rank, etc), as well as eigenvalues, eigenvectors, and their properties. Then we will move on to study special matrices and their decompositions, along with similarities, and Jordan canonical forms. In the third segment, we will define norms on vectors and matrices and study their analytic properties. Finally, we will discuss another important class of matrices - positive definite and semidefinite matrices. If time permits, we will also cover positive and negative matrices and their properties.

**Requirements/Evaluation:** Homework assignments and exams.

**Prerequisites:** Math 350/351 or permission of instructor

**Enrollment Limit:** 25

**Enrollment Preferences:** Mathematics and Statistics Majors, Seniors

**Expected Class Size:** 10

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This is an advanced mathematics class that covers complex properties of matrices and some of their applications.

Not offered current academic year

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**MATH 419 (F) Algebraic Number Theory** (QFR)

We all know that integers can be factored into prime numbers and that this factorization is essentially unique. In more general settings, it often still makes sense to factor numbers into "primes," but the factorization is not necessarily unique! This surprising fact was the downfall of Lamé's attempted proof of Fermat's Last Theorem in 1847. Although a valid proof was not discovered until over 150 years later, this error gave rise to a new branch of mathematics: algebraic number theory. In this course, we will study factorization and other number-theoretic notions in more abstract algebraic settings, and we will see a beautiful interplay between groups, rings, and fields.

**Requirements/Evaluation:** homework assignments and exams

**Prerequisites:** MATH 355, or permission of instructor

**Enrollment Limit:** 25

**Expected Class Size:** 20

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

Not offered current academic year

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**MATH 426 (F) Differential Topology** (QFR)

Differential topology marries the rubber-like deformations of topology with the computational exactness of calculus. This sub eld of mathematics asks
and answers questions like "Can you take an integral on the surface of a doughnut?" and includes far-reaching applications in relativity and robotics.

This tutorial will provide an elementary and intuitive introduction to differential topology. We will begin with the definition of a manifold and end with a generalized understanding of Stokes Theorem.

**Requirements/Evaluation:** weekly homework and exams, and possibly student presentations

**Prerequisites:** MATH 350 (students who have not taken MATH 350 may enroll only with permission of the instructor)

**Enrollment Limit:** 30

**Enrollment Preferences:** mathematics seniors, mathematics majors

**Expected Class Size:** 10

**Grading:** no pass/fail option, no fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** There will be weekly math problem sets.

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**MATH 427 (S) Tiling Theory** (QFR)

Since people first used stones and bricks to tile the floors of their domiciles, tiling has been an area of interest. Practitioners include artists, engineers, designers, architects, crystallographers, scientists and mathematicians. This course will be an investigation into the mathematical theory of tiling. The course will focus on tilings of the plane, including topics such as the symmetry groups of tilings, types of tilings, random tilings, the classification of tilings and aperiodic tilings. We will also look at tilings of the sphere, tilings of the hyperbolic plane, and tilings in in higher dimensions, including "knotted tilings".

**Requirements/Evaluation:** problem assignments, exams and a presentation/paper

**Prerequisites:** MATH 250 Linear Algebra and MATH 355 Abstract Algebra

**Enrollment Limit:** 30

**Enrollment Preferences:** senior majors, seniors, juniors, sophomores, first-year students (this is a senior seminar, one of which is required for all senior majors, so they have first preference)

**Expected Class Size:** 25

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** Mathematics course

Not offered current academic year

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**MATH 434 (S) Applied Dynamics and Optimal Control** (QFR)

We seek to understand how dynamical systems evolve, how that evolution depends on the various parameters of the system, and how we might manipulate those parameters to optimize an overall outcome. The primary focus of this course will be optimal control using Pontryagin's maximum principle and Hamilton-Jacobi-Bellman equations. These tools have broad application in ecology, economics, finance, and engineering, and we will draw on basic models from these fields to motivate our study. The course will begin with a solid review of modeling with dynamical systems, and deepening our understanding of differential and difference equations, parameter dependence, and bifurcations.

**Requirements/Evaluation:** exams, homework assignments, and projects

**Prerequisites:** MATH 309 or PHYS 210, and MATH 350 or 351, or permission of the instructor

**Enrollment Limit:** 25

**Enrollment Preferences:** Preference will be given to senior math majors.

**Expected Class Size:** 20

**Grading:** no pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This is a 400 level math course.
MATH 442 (F) Introduction to Descriptive Set Theory (QFR)

Descriptive set theory (DST) combines techniques from analysis, topology, set theory, combinatorics, and other areas of mathematics to study definable (typically Borel) subsets of Polish spaces. The first part of this course will cover the topics necessary to understand the main objects of study in DST: we will develop comfort with point-set topology (enough to juggle with Polish spaces and Borel sets), and set theory (just well-orderings and cardinality). The second part of the course will feature selected topics in descriptive set theory: for example, trees, the perfect set property, Baire category, and infinite games.

Requirements/Evaluation: Evaluation based on homework, exams, and classroom participation.

Prerequisites: Math 250 - Linear Algebra, Math 350/351 Real Analysis/Applied Real Analysis

Enrollment Limit: 14

Enrollment Preferences: Senior Math Majors, then non-Senior Math Majors

Expected Class Size: 14

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: Mathematics course in descriptive set theory.

Not offered current academic year

MATH 443 Introduction to Optimal Transport Theory (QFR)

This course will introduce you to the fascinating world of transportation optimization, a field that has important applications in many areas of science and engineering, such as economics, image processing, and machine learning. We will start by exploring the discrete Optimal Transport (OT) problem, which involves finding the most efficient way to transport a set of objects from one location to another. While the discrete OT problem can be formulated as a linear programming problem, finding an optimal solution to this problem can be computationally expensive, especially for large-scale problems. To overcome this computational challenge, a popular approach is to use entropy regularization. We will also investigate the entropy regularized OT problem, which provides us with an approximation of optimal transport, with lower computational complexity and easy implementation.

In the second half of the course, we will delve into the continuous case, which allows us to consider transport between infinitely many locations. We will study the famous Monge-Kantorovich problem, which involves finding the optimal transportation plan that minimizes the total cost of moving a given amount of mass from one location to another, subject to various constraints. Throughout the course, we will use a combination of theoretical and practical approaches to understand and apply the concepts we cover. By the end of the course, you will have a strong foundation in OT theory, which will prepare you for further studies in this exciting and rapidly evolving field. Recommended Textbooks / Articles: Topics in Optimal Transportation - Cédric Villani Optimal Transport for Applied Mathematicians - Filippo Santambrogio Computational Optimal Transport - Gabriel Peyré, Marco Cuturi


Prerequisites: Math 350/351 or permission of instructor

Enrollment Limit: 25

Enrollment Preferences: Mathematics and Statistics majors

Expected Class Size: 20

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is a senior seminar course in mathematics and will require students to use advanced quantitative and formal reasoning skills.

Not offered current academic year

MATH 445 (S) Topics in Numerical Analysis (QFR)

Numerical analysis is a field of mathematics that focuses on developing algorithms and computational methods to solve problems that cannot be solved exactly. In this senior seminar course on numerical analysis we will cover advanced topics such as numerical solutions of Partial Differential Equations, Random Numbers and Monte Carlo simulation, Fast Fourier Transform and signal processing, as well as applications or the Singular Value
Decomposition for matrices. The course will start with a review of basic concepts from calculus, linear algebra, and differential equations. Students who have taken Introduction to Numerical Analysis (Math 345) are welcome to take this course.

Requirements/Evaluation: exams and homework assignments
Prerequisites: Math 309 or Math 345 or permission of instructor
Enrollment Limit: 25
Enrollment Preferences: Mathematics Majors, Seniors
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This is a senior seminar course in mathematics.

MATH 453 (S) Partial Differential Equations (QFR)

In this course, we further explore the world of differential equations. Mainly, we cover topics in partial differential equations. Partial Differential Equations (PDEs) are fundamental to the modeling of many natural phenomena, arising in many fields, including fluid mechanics, heat and mass transfer, electromagnetic theory, finance, elasticity, and more. The goals of this course are to discuss the following topics: classification of PDEs in terms of order, linearity and homogeneity; physical interpretation of canonical PDEs; solution techniques, including separation of variables, series solutions, integral transforms, and the method of characteristics.

Requirements/Evaluation: Evaluation will be based on homework, projects, and exams.
Prerequisites: MATH 150-151; MATH/PHYS 210 or MATH 309
Enrollment Limit: 20
Enrollment Preferences: Mathematics and Physics majors.
Expected Class Size: 15
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This is an advanced mathematics class dedicated to the study of partial differential equations (PDEs). These equations are the most important mathematical tools for the study of complex physical phenomena such as waves and fluids (including both air and water), heat transfer, electromagnetism, and finance.

Spring 2025
LEC Section: 01 TR 11:20 am - 12:35 pm Bhagya Athukorallage

MATH 457 (S) Partition Theory (QFR)

The partitions of a positive integer are the different ways of writing it as a sum of positive integers. For example, 5 has seven partitions, three of which are 5=1+1+1+1+1, 5=2+3, and 5=5. (Can you find the rest?) Partition theory is a rich area of combinatorics with applications to algebra and mathematical physics. In this class we will focus on enumerative and bijective methods to answer questions such as: How can we calculate the number of partitions of a number efficiently? Why is the number of partitions of N into strictly odd numbers always the same as the number of its partitions into distinct numbers? Why does a 2-dimensional partition look like a stack of cubes, and what does that have to do with tilings?

Requirements/Evaluation: Written homework; Written/Oral Exams; Project/Presentation
Prerequisites: A course in abstract algebra such as MATH 355, or permission of instructor.
Enrollment Limit: 25
Enrollment Preferences: Priority given to Junior and Seniors, and according to previous experience with subject.
Expected Class Size: 10
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This is an advanced course in mathematics.
MATH 481 (S) Measure theory and Hilbert spaces (QFR)

How large is the unit square? One might measure the number of individual points in the square (uncountably infinite), the area of the square (1), or the dimension of the square (2). But what about for more complicated sets, e.g., the set of all rational points in the unit square? What's the area of this set? What's the dimension? In this course we'll come up with precise ways to measure size—length, area, volume, dimension, etc.—that apply to a broad array of sets. Along the way we'll encounter Lebesgue measure and Lebesgue integration, Hausdorff measure and fractals, space-filling curves and the Banach-Tarski paradox. We will also investigate Hilbert spaces, mathematical objects that combine the tidiness of linear algebra with the power of analysis and are fundamental to the study of differential equations, functional analysis, harmonic analysis, and ergodic theory, and also apply to fields like quantum mechanics and machine learning. This material provides excellent preparation for graduate studies in mathematics, statistics and economics.

Requirements/Evaluation: Problem sets, exams, an expository essay

Prerequisites: At least one previous course that has Math 350 or 351 as a prerequisite (e.g., Math 374, 383, 401, 404, 408, 420, 426, 485), or permission of instructor.

Enrollment Limit: 20

Enrollment Preferences: Juniors and seniors.

Expected Class Size: 10

Grading: no pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: It's math.

Not offered current academic year

MATH 493 (F)(S) Senior Honors Thesis: Mathematics

Mathematics senior honors thesis; this is part of a full-year thesis (493-494). Each student carries out an individual research project under the direction of a faculty member that culminates in a thesis. See description under The Degree with Honors in Mathematics.

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3)

Fall 2024
HON Section: 01 TBA Cesar E. Silva

Spring 2025
HON Section: 01 TBA Cesar E. Silva

MATH 494 (S) Senior Honors Thesis: Mathematics

Mathematics senior honors thesis; this is part of a full-year thesis (493-494). Each student carries out an individual research project under the direction of a faculty member that culminates in a thesis. See description under The Degree with Honors in Mathematics.

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3)

Not offered current academic year

MATH 497 (F)(S) Independent Study: Mathematics

Directed 400-level independent study in Mathematics.

Prerequisites: permission of department

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3)
MATH 498 (S) Independent Study: Mathematics
Directed 400-level independent study in Mathematics.
Prerequisites: permission of department
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)
Not offered current academic year

MATH 499 (F)(S) Senior Colloquium
Mathematics senior colloquium. Meets every week for two hours both fall and spring. Senior majors must participate at least one hour a week. This colloquium is in addition to the regular four semester-courses taken by all students.
Class Format: colloquium
Grading: non-graded
Distributions: No divisional credit

Winter Study
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MATH 12 (W) The Mathematics of LEGO and Outreach Activities
This course is a modification of nine previous winter studies I have done on the Mathematics of LEGO bricks and outreach activities. Similar to those, we will use LEGO bricks as a motivator to talk about some good mathematics (combinatorics, algorithms, efficiency). We will partner with local elementary schools to engage students and teachers in mathematics, as well as possibly prepare articles for journals. Possible activities include submitting a Lego Ideas Challenge, to try and create a set that Lego will then market and sell, or do a speed build challenge (college teams vs elementary school teams perhaps).

Requirements/Evaluation: short paper and final project or presentation
Prerequisites: none
Enrollment Limit: 30
Enrollment Preferences: none
Expected Class Size: NA
Grading: pass/fail only
Materials/Lab Fee: $195
Attributes: EXPE Experiential Education Courses SLFX Winter Study Self-Expression
Not offered current academic year

MATH 14 (W) Flow Yoga
This class will teach the practice of yoga based on a fusion of Ashtanga, Vinyasa and Iyengar methodologies. This course is open to all levels from those who have never done a pose to those who have practicing yoga for years. The students will learn how to connect movement to breath in a sequence of vigorous yoga poses, or asanas. The flow classes will introduce yoga sequences in an alignment-based curriculum appropriate to each student's experience. In addition to the physical work out of practicing yoga, there will be short discussions on the assigned reading of handouts that explore various topics pertinent to yoga, such as an introduction to yogic philosophy, the historical background of yoga, and a basic of knowledge of Sanskrit. The rudiments of Meditation will begin and end every session. Attendance to every class is a requirement. We will meet 3 times per week for 2 hours each time. The students will be expected to practice yoga daily on their own after having memorized certain basic yoga sequences. Each student will keep a journal where they will note the highlight of that day's class, whether it be a new asana or a sutra or whatever caught their attention. Students will be evaluated on their class participation, their journal and a short essay on a topic of their choice.

Requirements/Evaluation: Class participation and journal entries and final essay

Prerequisites: Those with previous yoga experience

Enrollment Limit: 25

Enrollment Preferences: 15-20

Expected Class Size: NA

Grading: pass/fail only

Materials/Lab Fee: $106

Attributes: WELL Winter Study Wellness

Not offered current academic year

MATH 16 Translating Akhmatova

Anna Akhmatova is one of the most celebrated Russian poets of the 20th century. Our goal in this course will be to read and translate some of her work, in particular the powerful *Requiem*. Proficiency (but not fluency!) in Russian is required, as is an interest in poetry and translation.

Class Format: The plan is to meet 9 hours per week (3 hours each on Mondays, Wednesdays, Fridays) to read poetry aloud and collaborate on translations.

Requirements/Evaluation: We will read aloud to one another, as well work on translations collaboratively.

Prerequisites: Proficiency in Russian.

Enrollment Limit: 6

Enrollment Preferences: Russian speakers who love poetry.

Expected Class Size: 3

Grading: Not offered current academic year

MATH 21 Up-cycling of Waste Cotton into Bioplastics

The pressing concerns of plastic pollution have spurred considerable interest in biomass-based products sourced from sustainable and renewable origins, offering attributes such as recyclability, compostability, or biodegradability. Cotton, being the purest form of cellulose found in nature, presents a promising avenue for exploration. Annually, the cotton industry generates substantial quantities of gin waste and low-quality cotton fibers, typically unsuitable for fabric production. The course will provide students with an overview of developing bioplastic materials (i.e., films and aerogels), especially from waste cotton, to effectively curtail the carbon footprint and mitigate the detrimental impacts of synthetic plastics. We will begin by addressing critical issues surrounding plastic pollution before delving into interesting topics such as bioplastics; cotton fibers (journey from field to fabric); chemical insight into cellulose; transforming waste cotton into bioplastics through dissolution; and functionalization, characterization, and applications of bioplastics. Additionally, a brief overview of water-soluble bioplastics and their benefits and applications will be provided. Students will demonstrate their understanding through class discussions and will either deliver a 10-minute class presentation and submit a 5-page report, exploring and analyzing the ideas and topics covered during the lectures.

Requirements/Evaluation: Class participation; 5-page paper and a presentation

Prerequisites: CHEM 101 or permission of instructor

Enrollment Limit: 15

Enrollment Preferences: The instructor aims to attract a diverse group of students interested in acquiring foundational knowledge in the
development of bioplastics from sustainable natural resources

Expected Class Size: 15

Grading:

Unit Notes: Sumedha Liyanage has academic research expertise on converting plant-based polymers into bioplastics, specifically targeting various industrial applications and has a Ph.D. in Plant and Soil Science (Fiber & Biopolymers and FTIR imaging)

Not offered current academic year

MATH 22  Stoppard and Go: Interpretation and Scenework for Three of Tom Stoppard's Plays

Tom Stoppard has been cited at one of the greatest living playwrights, with numerous prestigious awards, frequent international productions of his extensive oeuvre, and a knighthood from Queen Elizabeth. "Stoppardian" has even become a term describing works that use wit and comedy while exploring philosophical concepts, succinctly expressing his interdisciplinary appeal. We will investigate three of Stoppard's most prominent plays: *Rosencrantz and Guildenstern are Dead* (1967); *Travesties* (1975); and *Arcadia* (1993). The course will give students the chance to engage with the texts through discussion, collaborative scenework, and informal performances. Each play invites us to consider the past and the present anew through humor and unexpected juxtapositions: R&G reframes Shakespeare's *Hamlet* through formerly minor characters; *Travesties* portrays the convergence in 1917 Zurich of Dada founder, Tristan Tzara, author James Joyce, and the communist revolutionary, Lenin; *Arcadia* explores the thorniness of attribution and biography in literature, scholarship and science. Intertwining themes of history, art, literature, politics, aesthetics, mathematics, and philosophy, these plays will amuse and challenge students of all academic backgrounds and will offer a rich launchpad for our discussion and workshop-focused class sessions. Brief supplementary readings will help ground students in the historic and literary setting of each play, while film viewings, in-class scene work, and informal performances will bring the language to life. No theatrical experience necessary, but workingshopping and presenting scenes with and to classmates is a required component of this course.

Requirements/Evaluation:  
Presentation(s); Other: participation in class discussion and workingshopping scenes

Prerequisites: None

Enrollment Limit: 15

Enrollment Preferences: Preference will be given to juniors and seniors

Expected Class Size: NA

Grading:

Unit Notes: Amanda Bell Goldmakher is an arts educator and administrator who has worked at nonprofits in NYC, MA and DC, taught in the Education Department at the Clark Art Museum, and led local theater workshops and youth productions.

Materials/Lab Fee: $60

Attributes:  
EXPE Experiential Education Courses  
SLFX Winter Study Self-Expression

Not offered current academic year

MATH 29  Math Enrichment in Public Schools

We will explore math education and enrichment at all levels. Class will include a combination of lecture and discussion in a traditional classroom setting at Williams. Students will also teach lessons at the Williamstown Elementary School math club and Adventures in Learning series throughout the month of January. We will likely have one event with the Mt Greylock high school math club as well. We will present and teach various math enrichment topics including primes, Fibonacci numbers, infinity, the math of voting, fair division, logic, and more. Some proof writing experience is recommended, see instructor if you're not sure you satisfy the pre-requisites.

Class Format: Class will include lecture, and teaching/assisting at local schools. We will be primarily at the elementary school, but will likely have an event at the high school as well.

Requirements/Evaluation:  Students will be evaluated on their teaching performance and preparation, teaching journal, and final paper.

Prerequisites: Some experience proof writing. Math 200 or 250 or permission of instructor.

Enrollment Limit: 10

Enrollment Preferences: Preference given to those with interest in mathematics and education. Course enrollment will be determined by answers to a questionnaire if over-enrolled.

Expected Class Size: 8

Grading:
Materials/Lab Fee: $100 for books and materials
Not offered current academic year

MATH 30 (W) Senior Project: Mathematics
To be taken by candidates for honors in Mathematics other than by thesis route.
Class Format: honors project
Grading: pass/fail only
Not offered current academic year

MATH 31 (W) Senior Thesis: Mathematics
To be taken by students registered for Mathematics 493-494.
Class Format: thesis
Grading: pass/fail only
Not offered current academic year

MATH 99 (W) Independent Study: Mathematics
Open to upperclass students. Students interested in doing an independent project (99) during Winter Study must make prior arrangements with a faculty sponsor. The student and professor then complete the independent study proposal form available online. The deadline is typically in late September. Proposals are reviewed by the pertinent department and the Winter Study Committee. Students will be notified if their proposal is approved prior to the Winter Study registration period.
Class Format: independent study
Grading: pass/fail only
Not offered current academic year