MATHEMATICS AND STATISTICS (Div III)

MATHEMATICS

Chair: Professor Susan Loepp


On leave Fall/Spring: Professors: B. Klingenberg, M. Stoiciu.
On leave Fall only: Professor A. Pacelli.
On leave Spring only: Associate Professor S. Miller.

MAJOR IN MATHEMATICS

The major in Mathematics is designed to meet two goals: to introduce some of the central ideas in a variety of areas of mathematics and statistics, and to develop problem-solving ability by teaching students to combine creative thinking with rigorous reasoning. Mathematics is a gateway to many career paths including statistics, teaching, consulting, business, engineering, finance, actuarial studies and applied mathematics. Students are strongly encouraged to consult with the department faculty on choosing courses appropriate to an individualized program of study.

REQUIREMENTS (9 courses plus colloquium)

The major in Mathematics consists of nine courses plus the colloquium requirement. Mathematics is highly cumulative, and students should plan a route to completing the major that ensures the proper sequencing and prerequisites for all needed courses. Note that not all upper level courses are offered every year.

Calculus (2 courses)

Mathematics 140 Calculus II
Mathematics 150 or 151 Multivariable Calculus

Applied/Discrete Mathematics/Statistics (1 course)

Mathematics 209 Differential Equations
or Mathematics 210 Mathematical Methods for Scientists (Same as Physics 210)
or Mathematics 200 Discrete Mathematics
or Statistics 201 Statistics and Data Analysis
or Statistics 231 Statistical Design of Experiments
or a more advanced applied/discrete/statistics course with prior department approval

Core Courses (3 courses)

Mathematics 250 Linear Algebra
Mathematics 350 Real Analysis or Mathematics 351 Applied Real Analysis
Mathematics 355 Abstract Algebra

Completion (3 courses plus colloquium)

Two electives from courses numbered 300 and above, or STAT 231

One Senior Seminar: Any course numbered between 400 and 479, taken in the senior year.

Participation in the Department Colloquium, in which all senior majors present and attend talks on mathematical or statistical topics of their choice. Majors have to attend at least 20 colloquia in their senior year and present one themselves.

ADVANCED PLACEMENT

Students who come to Williams with advanced placement will be moved up in the Mathematics major, and should consult with faculty to be placed
in the best class reflecting their experience and background. A student who places out of a course substitutes another course of equal or higher level in Mathematics or Statistics to complete the nine course major. Students should select courses best suited to their preparation and goals, and consult with the department faculty concerning appropriate courses and placement. The department reserves the right to refuse registration in any course for which the student is overqualified.

For Example, a student starting in MATH 130 might take MATH 130 and 140 the first year, MATH 150 and MATH 200 the second year, MATH 250 and MATH 350 the third year, MATH 355 and a senior seminar the fourth year, plus the two required electives some time. Students are encouraged to consult freely with any math faculty about course selection and anything else.

**CALCULUS PLACEMENT**

**Recommended placement for students who have taken an Advanced Placement Examination in Calculus (AB or BC) is:**

- BC 1, 2 or AB 2, 3 Math 140
- AB 4 or 5 Math 150
- BC 3, 4 or 5 Math 151

Consult with department faculty for any Calculus or Statistics placement questions. Students who have had calculus in high school, whether or not they took the Advanced Placement Examination, are barred from 130 unless they obtain permission from the instructor.

**NOTES**

Substitutions, Study Abroad, and Transfer Credit: In some cases, and with prior permission of the Mathematics and Statistics Department, appropriate courses from other institutions or a course from another Williams department may be substituted for electives. Programs like the “Budapest Semester in Mathematics” are recommended for majors who wish to focus on mathematics away. The department, though, normally accommodates students who select other study away programs. The department offers its core courses in both the fall and the spring to allow students to spend more easily a semester away.

**Double Counting:** No course may count towards two different majors.

**Early Senior Seminar:** The senior seminar is designed and intended to be taken during the senior year. Students who have made significant progress towards the major may request to fulfill this requirement with a senior seminar taken during their junior year. Such requests should be submitted to the department chair, and should include a plan for completing the major and the rationale for taking their senior seminar as a Junior.

**Planning Courses:** Core courses Mathematics 350, 351, 355, and Statistics 346 are normally offered every year. Most other 300-level topics are offered in alternate years. Topology, Complex Analysis, and second courses in real analysis and abstract algebra are normally offered at least every other year.

Each 400-level topic is normally offered every two to four years. Students should check with the department before planning far into the future.

**Course Admission:** Courses are normally open to all students meeting the prerequisites, subject to any course caps. Students with questions about the level at which courses are conducted are invited to consult department faculty.

**FAQ**

Students MUST contact departments/programs BEFORE assuming study away credit will be granted toward the major or concentration.

**Can your department or program typically pre-approve courses for major/concentration credit?**

Yes, in many cases, though students should be sure to contact the department.

**What criteria will typically be used/required to determine whether a student may receive major/concentration credit for a course taken while on study away?**

- Course title and description, and complete syllabus including readings/assignments.

**Does your department/program place restrictions on the number of major/concentration credits that a student might earn through study away?**

- No.

**Does your department/program place restrictions on the types of courses that can be awarded credit towards your major?**

- Yes. They have to be approved MATH/STAT courses.

**Are there specific major requirements that cannot be fulfilled while on study away?**

- Yes. Colloquium requirement, Senior Seminar requirement.

**Are there specific major requirements in your department/program that students should be particularly aware of when weighing study away options?** (Some examples might include a required course that is always taught in one semester, laboratory requirements.)
Yes. The highly cumulative structure of the major.

Give examples in which students thought or assumed that courses taken away would count toward the major or concentration and then learned they wouldn’t:

None to date.

THE DEGREE WITH HONORS IN MATHEMATICS

The degree with honors in Mathematics is awarded to the student who has demonstrated outstanding intellectual achievement in a program of study which extends beyond the requirements of the major. The principal considerations for recommending a student for the degree with honors will be: Mastery of core material and skills, breadth and, particularly, depth of knowledge beyond the core material, ability to pursue independent study of mathematics or statistics, originality in methods of investigation, and, where appropriate, creativity in research.

An honors program normally consists of two semesters (MATH/STAT 493 and 494) and a winter study (WSP 031) of independent research, culminating in a thesis and a presentation. Under certain circumstances, the honors work can consist of coordinated study involving a one semester (MATH/STAT 493 or 494) and a winter study (WSP 030) of independent research, culminating in a “minithesis” and a presentation. At least one semester should be in addition to the major requirements, and thesis courses do not count as 400-level senior seminars.

An honors program in actuarial studies requires significant achievement on four appropriate examinations of the Society of Actuaries.

Highest honors will be reserved for the rare student who has displayed exceptional ability, achievement or originality. Such a student usually will have written a thesis, or pursued actuarial honors and written a mini-thesis. An outstanding student who writes a mini-thesis, or pursues actuarial honors and writes a paper, might also be considered. In all cases, the award of honors and highest honors is the decision of the Department.

APPLIED MATHEMATICS TRACK

Students interested in applied mathematics, engineering, or other sciences should consider:

Mathematics 140 Calculus II
Mathematics 150 or 151 Multivariable Calculus
Statistics 201 Statistics and Data Analysis
Mathematics 250 Linear Algebra
Mathematics 351 Applied Real Analysis
Mathematics 355 Abstract Algebra
Some programming or numerical analysis (e.g. MATH 361, 318T, or anything if you’ve had CSCI 134)
MATH 309 or Post-core Differential Equations/Numerical Methods
Senior seminar (e.g. Math Ecology MATH 410T or Mathematical Modeling MATH 433)

Other recommended courses: complex analysis, discrete mathematics (e.g. combinatorics or graph theory), operations research, optimization, probability, statistics, appropriate courses in Biology, Chemistry, Computer Science, Economics, Neuroscience, Physics, etc.

Williams has exchange and joint programs with good engineering schools. Interested students should consult the section on engineering near the beginning of the Bulletin and the Williams pre-engineering advisor for further information.

GRADUATE SCHOOL TRACK

Students interested in continuing their study of mathematics in graduate school should consider:

Mathematics 140 Calculus II
Mathematics 150 or 151 Multivariable Calculus
Mathematics 250 Linear Algebra
Mathematics 350 Real Analysis
Mathematics 355 Abstract Algebra
Complex Analysis
Topology
Some second semester analysis
Some second semester algebra
Some post-core geometry

Thesis

[With prior permission, in unusual circumstances, senior seminar can be waived in favor of harder post-core electives.]

Students headed for graduate school generally take more than this relatively small number of courses required for a liberal arts major. Reading knowledge of a foreign language (French, German, or Russian) can be helpful.

Students interested in continuing their study of statistics in graduate school should take STAT 201, 231, 346, a 400 level statistics course and MATH 350 and 341.

OTHER CAREER PATHS

Other Graduate and Professional Schools: An increasing number of graduate and professional schools require mathematics and statistics as a prerequisite to admission or to attaining their degree. Students interested in graduate or professional training in business, medicine, economics, or psychology are advised to find out the requirements in those fields early in their college careers.

Statistics and Actuarial Science: Students interested in statistics or actuarial science should consider Mathematics 341, Statistics courses, and Economics 255. Additionally, students should consider taking some number of the actuarial exams given by the Society of Actuaries, which can constitute part of an honors program in actuarial studies (see section on honors above).

Teaching: Students interested in teaching mathematics at the elementary or secondary school level should consider courses on teaching, number theory, geometry, statistics, and practice as a tutor or teaching assistant. Winter study courses that provide a teaching practicum are also highly recommended. Consult the Program in Teaching (Professor Susan Engel) and the Office of Career Counseling.

Business and Finance: Students interested in careers in business or finance should consider Mathematics 333 and Statistics courses. Since these courses address different needs, students should consult with the instructors to determine which seem to be most appropriate for individuals.

There are three types of 300-level courses. There are the core courses: Real Analysis, MATH 350/351, and Abstract Algebra Math 355. There are the "precore" courses, which do not have the core courses as prerequisites and have numbers 300-349. Finally, there are those courses that have an Abstract Algebra or Real Analysis prerequisite, which are numbered 360-399.

MATH 102 (F) Foundations in Quantitative Skills

This course is designed to strengthen the student's foundation in quantitative reasoning in preparation for the science curriculum and QFR requirements. The material will cover topics at the college algebra/precalculus level with a particular emphasis on the computational and applied side of mathematics. Access to this course is limited to placement by a quantitative skills counselor.

Class Format: lecture

Requirements/Evaluation: homework assignments, quizzes and class participation

Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option

Prerequisites: access to the course is limited to placement by a quantitative skills counselor

Enrollment Limit: 15

Enrollment Preferences: students who need most help with the quantitative reasoning

Expected Class Size: 10

Distributions: (D3)

Fall 2018

LEC Section: 01 MWF 9:00 am - 9:50 am Lori A. Pedersen

MATH 110 (F) Logic and Likelihood (QFR)

How best can we reason in the face of uncertainty? We will begin with an examination of rationality and the reasoning process including a survey of formal logic. Starting with uncertainty from a psychological and philosophical viewpoint, we will move to a careful theory of likelihood and how to reason with probabilistic models. The course will conclude with a consideration of observation and information, how to test hypotheses, and how we update our beliefs to incorporate new evidence.
Class Format: lecture

Requirements/Evaluation: homework, essays, presentations, exams, and participation

Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option

Prerequisites: none

Enrollment Limit: 25

Enrollment Preferences: first-year students

Expected Class Size: 25

Distributions: (D3) (QFR)

Distribution Notes: QFR: This course will be covering formal logic and probability theory at sufficient depth to place this course on level with other QFR designated courses.

Fall 2018

LEC Section: 01    MWF 11:00 am - 11:50 am     Stewart D. Johnson

MATH 113 (F) The Beauty of Numbers (QFR)

Have you ever wondered what keeps your credit card information safe everytime you buy something online? Number theory! Number Theory is one of the oldest branches of mathematics. In this course, we will discover the beauty and usefulness of numbers, from ancient Greece to modern cryptography. We will look for patterns, make conjectures, and learn how to prove these conjectures. Starting with nothing more than basic high school algebra, we will develop the logic and critical thinking skills required to realize and prove mathematical results. Topics to be covered include the meaning and content of proof, prime numbers, divisibility, rationality, modular arithmetic, Fermat's Last Theorem, the Golden ratio, Fibonacci numbers, coding theory, and unique factorization.

Class Format: lecture

Requirements/Evaluation: evaluation will be based primarily on projects, homework assignments, and exams

Prerequisites: MATH 102 (or demonstrated proficiency on a diagnostic test) or permission of instructor

Enrollment Limit: 25

Expected Class Size: 25

Distributions: (D3) (QFR)

Not offered current academic year

MATH 130 (F) Calculus I (QFR)

Calculus permits the computation of velocities and other instantaneous rates of change by a limiting process called differentiation. The same process also solves "max-min" problems: how to maximize profit or minimize pollution. A second limiting process, called integration, permits the computation of areas and accumulations of income or medicines. The Fundamental Theorem of Calculus provides a useful and surprising link between the two processes. Subtopics include trigonometry, exponential growth, and logarithms.

Class Format: lecture

Requirements/Evaluation: evaluation will be based primarily on exams, homework and quizzes

Prerequisites: MATH 102 (or demonstrated proficiency on a diagnostic test); this is an introductory course for students who have not seen calculus before

Enrollment Limit: 50

Enrollment Preferences: Professor's discretion

Expected Class Size: 30

Department Notes: students who have previously taken a calculus course may not enroll in MATH 130 without the permission of instructor

Distributions: (D3) (QFR)

Fall 2018
MATH 140 (F) Calculus II (QFR)

Mastery of calculus requires understanding how integration computes areas and business profit and acquiring a stock of techniques. Further methods solve equations involving derivatives ("differential equations") for population growth or pollution levels. Exponential and logarithmic functions and trigonometric and inverse functions play an important role. This course is the right starting point for students who have seen derivatives, but not necessarily integrals, before.

Class Format: lecture

Requirements/Evaluation: evaluation will be based primarily on homework, quizzes, and/or exams

Prerequisites: MATH 130 or equivalent; students who have received the equivalent of advanced placement of AB 4, BC 3 or higher may not enroll in MATH 140 without the permission of instructor

Enrollment Limit: 50

Expected Class Size: 30

Department Notes: students who have higher advanced placement must enroll in MATH 150 or above

Distributions: (D3) (QFR)

MATH 150 (F) Multivariable Calculus (QFR)

Applications of calculus in mathematics, science, economics, psychology, the social sciences, involve several variables. This course extends calculus to several variables: vectors, partial derivatives, multiple integrals. There is also a unit on infinite series, sometimes with applications to differential equations.

Class Format: lecture

Requirements/Evaluation: evaluation will be based primarily on homework, quizzes, and/or exams

Prerequisites: MATH 140 or equivalent, such as satisfactory performance on an Advanced Placement Examination

Enrollment Limit: 50

Expected Class Size: 50

Department Notes: this course is the right starting point for students who have seen differentiation and integration before; students with the equivalent of advanced placement of AB 4, BC 3 or above should enroll in MATH 150

Distributions: (D3) (QFR)
MATH 151 (F) Multivariable Calculus  (QFR)
Applications of calculus in mathematics, science, economics, psychology, the social sciences, involve several variables. This course extends calculus to several variables: vectors, partial derivatives and multiple integrals. The goal of the course is Stokes Theorem, a deep and profound generalization of the Fundamental Theorem of Calculus. The difference between this course and MATH 150 is that MATH 150 covers infinite series instead of Stokes Theorem. Students with the equivalent of BC 3 or higher should enroll in MATH 151, as well as students who have taken the equivalent of an integral calculus and who have already been exposed to infinite series. For further clarification as to whether MATH 150 or MATH 151 is appropriate, please consult a member of the math/stat department.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on homework, quizzes, and/or exams
Prerequisites: AP BC 3 or higher or integral calculus with infinite series
Enrollment Limit: 50
Expected Class Size: 50
Department Notes: MATH 151 satisfies any MATH 150 prerequisite; credit will not be given for both MATH 150 and MATH 151
Distributions: (D3) (QFR)

Fall 2018
LEC Section: 01 MWF 9:00 am - 9:50 am Colin C. Adams
LEC Section: 02 MWF 10:00 am - 10:50 am Colin C. Adams
LEC Section: 03 MWF 11:00 am - 11:50 am Colin C. Adams

MATH 200 (F) Discrete Mathematics  (QFR)
Course Description: In contrast to calculus, which is the study of continuous processes, this course examines the structure and properties of finite sets. Topics to be covered include mathematical logic, elementary number theory, mathematical induction, set theory, functions, relations, elementary combinatorics and probability, graphs and trees, and algorithms. Emphasis will be given on the methods and styles of mathematical proofs, in order to prepare the students for more advanced math courses.

Class Format: lecture/discussion
Requirements/Evaluation: evaluation will be based primarily on homework and exams
Prerequisites: MATH 140 or MATH 130 with CSCI 134 or 135; or one year of high school calculus with permission of instructor; students who have taken a 300-level math course should obtain permission of the instructor before enrolling
Enrollment Limit: 40
Expected Class Size: 25
Distributions: (D3) (QFR)

Fall 2018
LEC Section: 01 MWF 9:00 am - 9:50 am Ralph E. Morrison
LEC Section: 02 MWF 10:00 am - 10:50 am Ralph E. Morrison

Spring 2019
LEC Section: 01 MWF 10:00 am - 10:50 am Leo Goldmakher
LEC Section: 02 MWF 11:00 am - 11:50 am Leo Goldmakher

MATH 209 (S) Differential Equations  (QFR)
Historically, much beautiful mathematics has arisen from attempts to explain physical, chemical, biological and economic processes. A few ingenious techniques solve a surprisingly large fraction of the associated ordinary and partial differential equations, and geometric methods give insight to many more. The mystical Pythagorean fascination with ratios and harmonics is vindicated and applied in Fourier series and integrals. We will explore the methods, abstract structures, and modeling applications of ordinary and partial differential equations and Fourier analysis.
MATH 210 (S) Mathematical Methods for Scientists (QFR)

Crosslistings: PHYS210 / MATH210

Secondary Crosslisting

This course covers a variety of mathematical methods used in the sciences, focusing particularly on the solution of ordinary and partial differential equations. In addition to calling attention to certain special equations that arise frequently in the study of waves and diffusion, we develop general techniques such as looking for series solutions and, in the case of nonlinear equations, using phase portraits and linearizing around fixed points. We study some simple numerical techniques for solving differential equations. A series of optional sessions in Mathematica will be offered for students who are not already familiar with this computational tool.

Class Format: lecture, three hours per week

Requirements/Evaluation: evaluation will be based on several exams and on weekly problem sets, all of which have a substantial quantitative component

Prerequisites: MATH 150 or 151 and familiarity with Newtonian mechanics at the level of PHYS 131

Enrollment Limit: 50

Expected Class Size: 30

Distributions: (D3) (QFR)

Spring 2019

LEC Section: 01    TR 9:55 am - 11:10 am     Daniel P. Aalberts

MATH 250 (F) Linear Algebra (QFR)

Many social, political, economic, biological, and physical phenomena can be described, at least approximately, by linear relations. In the study of systems of linear equations one may ask: When does a solution exist? When is it unique? How does one find it? How can one interpret it geometrically? This course develops the theoretical structure underlying answers to these and other questions and includes the study of matrices, vector spaces, linear independence and bases, linear transformations, determinants and inner products. Course work is balanced between theoretical and computational, with attention to improving mathematical style and sophistication.

Class Format: lecture

Requirements/Evaluation: evaluation will be based primarily on homework and exams

Prerequisites: MATH 150/151 or MATH 200

Enrollment Limit: 45

Expected Class Size: 35

Distributions: (D3) (QFR)

Attributes: COGS Related Courses;

Fall 2018

LEC Section: 01    TF 1:10 pm - 2:25 pm     Haydee M. A. Lindo

LEC Section: 02    TF 2:35 pm - 3:50 pm     Haydee M. A. Lindo

Spring 2019

LEC Section: 01    MWF 10:00 am - 10:50 am     Thomas A. Garrity
MATH 293 (F) Undergraduate Research Topics in Representation Theory (QFR)
Central to the study of the representation theory of Lie algebras is the computation of weight multiplicities by using Kostant's weight multiplicity formula. This formula is an alternating sum over a finite group, and involves a partition function. In this tutorial, we will address questions regarding the number of terms contributing nontrivially to the sum and develop closed formulas for the value of the partition function. Techniques used include generating functions and counting arguments, which are at the heart of combinatorics and are accessible to undergraduate students.

Class Format: tutorial

Requirements/Evaluation: written assignments, oral presentations

Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option

Prerequisites: permission of instructor

Enrollment Limit: 10

Enrollment Preferences: programming experience, students with interests in the intersection of combinatorics and abstract algebra

Expected Class Size: 10

Distributions: (D3) (QFR)

Not offered current academic year

MATH 306 (S) Fractals and Chaos (QFR)
Early in the course we introduce the notion of dynamical systems. Then we will develop the mathematics behind iterated function systems and study the notions of fractals and chaos. There will be a lot of computer experimentation with various programs and resources which the students are expected to use to learn and discover properties of fractals. The final topics will include dimension complex dynamics and the Mandelbrot set.

Class Format: lecture

Requirements/Evaluation: homework, projects and exams

Prerequisites: MATH 250

Enrollment Limit: 30

Expected Class Size: 18

Distributions: (D3) (QFR)

Not offered current academic year

MATH 307 (F) Computational Linear Algebra (QFR)
Linear algebra is of central importance in the quantitative sciences, including application areas such as image and signal processing, data mining, computational finance, structural biology, and much more. When the problems must be solved computationally, approximation, round-off errors, convergence, and efficiency matter, and traditional linear algebra techniques may fail to succeed. We will adopt linear algebra techniques on a large scale, implement them computationally, and apply them to core problems in scientific computing. Topics may include: systems of linear and nonlinear equations; approximation and statistical function estimation; optimization; interpolation; and Monte Carlo techniques. This course could also be considered a course in numerical analysis or computational science.

Class Format: lecture

Requirements/Evaluation: quizzes/exams, problem sets, projects and activities

Extra Info: may not be taken on a pass/fail basis

Prerequisites: Math 250, some elementary computer programming experience is strongly recommended

Enrollment Limit: 30

Enrollment Preferences: Professor's discretion

Expected Class Size: 25

Distributions: (D3) (QFR)
MATH 309 (S) Differential Equations  (QFR)
Ordinary differential equations (ODE) frequently arise as models of phenomena in the natural and social sciences. This course presents core ideas of ODE from an applied standpoint. Topics covered early in the course include numerical solutions, separation of variables, integrating factors, constant coefficient linear equations, and power series solutions. Later, we focus on nonlinear ODE, for which it is usually impossible to find analytical solutions. Tools from dynamical systems allows us to obtain some information about the behavior of the ODE without explicitly knowing the solution.

Class Format: lecture, discussion, interactive activities
Requirements/Evaluation: quizzes/exams, problem sets, activities
Extra Info: may not be taken on a pass/fail basis
Prerequisites: MATH 150/151 and MATH 250
Enrollment Limit: 30
Enrollment Preferences: Professor's discretion
Expected Class Size: 30
Distributions: (D3) (QFR)

Spring 2019
LEC Section: 01    TF 2:35 pm - 3:50 pm     Chad M. Topaz

MATH 310 (S) Mathematical Biology  (QFR)
Crosslistings: MATH310 / BIOL210
Primary Crosslisting
This course will provide an introduction to the many ways in which mathematics can be used to understand, analyze, and predict biological dynamics. We will learn how to construct mathematical models that capture essential properties of biological processes while maintaining analytic tractability. Analytic techniques, such as stability and bifurcation analysis, will be introduced in the context of both continuous and discrete time models. Additionally, students will couple these analytic tools with numerical simulation to gain a more global picture of the biological dynamics. Possible biological applications include, but are not limited to, single and multi-species population dynamics, neural and biological oscillators, tumor cell growth, and infectious disease dynamics.

Class Format: tutorial
Requirements/Evaluation: problem sets, weekly meetings, final project and paper
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 250, MATH 209 or 309, permission of instructor
Enrollment Limit: 10
Enrollment Preferences: if over-enrolled, will have students submit reasons for taking class; preference to those with interest in both subjects
Expected Class Size: 10
Distributions: (D3) (QFR)
Distribution Notes: QFR: The course will introduce methods for developing and analyzing mathematical models.
Attributes: PHLH Methods in Public Health;

Spring 2019
TUT Section: T1    TBA     Julie C. Blackwood

MATH 313 (F) Introduction to Number Theory  (QFR)
The study of numbers dates back thousands of years, and is fundamental in mathematics. In this course, we will investigate both classical and modern questions about numbers. In particular, we will explore the integers, and examine issues involving primes, divisibility, and congruences. We will also
look at the ideas of number and prime in more general settings, and consider fascinating questions that are simple to understand, but can be quite difficult to answer.

**Class Format:** lecture  
**Requirements/Evaluation:** evaluation will be based primarily on performance on homework, projects, and examinations  
**Prerequisites:** MATH 250 or permission of instructor  
**Enrollment Limit:** 40  
**Expected Class Size:** 25  
**Distributions:** (D3) (QFR)  

Fall 2018  
LEC Section: 01    MR 2:35 pm - 3:50 pm    Leo Goldmakher

MATH 316 (S) Protecting Information: Applications of Abstract Algebra and Quantum Physics  
**Crosslistings:** MATH316 / PHYS316  
**Primary Crosslisting**

Living in the information age, we find ourselves depending more and more on codes that protect messages against either noise or eavesdropping. This course examines some of the most important codes currently being used to protect information, including linear codes, which in addition to being mathematically elegant are the most practical codes for error correction, and the RSA public key cryptographic scheme, popular nowadays for internet applications. We also study the standard AES system as well as an increasingly popular cryptographic strategy based on elliptic curves. Looking ahead by a decade or more, we show how a quantum computer could crack the RSA scheme in short order, and how quantum cryptographic devices will achieve security through the inherent unpredictability of quantum events.

**Class Format:** lecture  
**Requirements/Evaluation:** evaluation will be based on homework sets and exams  
**Prerequisites:** PHYS/MATH 210 or MATH 250 (possibly concurrent) or permission of instructors;  
**Enrollment Limit:** 50  
**Expected Class Size:** 35  
**Department Notes:** students not satisfying the course prerequisites but who have completed MATH 200 or MATH 209 are particularly encouraged to ask to be admitted  
**Distributions:** (D3) (QFR)  

Not offered current academic year

MATH 318 (F) Numerical Problem Solving  
In the last twenty years computers have profoundly changed the work in numerical mathematics (in areas from linear algebra and calculus to differential equations and probability). The main goal of this tutorial is to learn how to use computers to do quantitative science. We will explore concepts and ideas in mathematics and science using numerical methods and computer programming. We will use specialized software, including Mathematica and Matlab.

**Class Format:** tutorial  
**Requirements/Evaluation:** evaluation will be based on homework, classwork, and exams  
**Extra Info:** may not be taken on a pass/fail basis; not available for the fifth course option  
**Prerequisites:** MATH 150/151 and MATH 250 or permission of instructor  
**Enrollment Limit:** 10  
**Expected Class Size:** 10  
**Distributions:** (D3) (QFR)  

Not offered current academic year
MATH 319 (F)  Integrative Bioinformatics, Genomics, and Proteomics Lab  (QFR)

Crosslistings: BIOL319 / CHEM319 / MATH319 / PHYS319 / CSCI319

Secondary Crosslisting

What can computational biology teach us about cancer? In this capstone experience for the Genomics, Proteomics, and Bioinformatics program, computational analysis and wet-lab investigations will inform each other, as students majoring in biology, chemistry, computer science, mathematics/statistics, and physics contribute their own expertise to explore how ever-growing gene and protein data-sets can provide key insights into human disease. In this course, we will take advantage of one well-studied system, the highly conserved Ras-related family of proteins, which play a central role in numerous fundamental processes within the cell. The course will integrate bioinformatics and molecular biology, using database searching, alignments and pattern matching, phylogenetics, and recombinant DNA techniques to reconstruct the evolution of gene families by focusing on the gene duplication events and gene rearrangements that have occurred over the course of eukaryotic speciation. By utilizing high throughput approaches to investigate genes involved in the MAPK signal transduction pathway in human colon cancer cell lines, students will uncover regulatory mechanisms that are aberrantly altered by siRNA knockdown of putative regulatory components. This functional genomic strategy will be coupled with independent projects using phosphorylation-state specific antisera to test our hypotheses. Proteomic analysis will introduce the students to de novo structural prediction and threading algorithms, as well as data-mining approaches and Bayesian modeling of protein network dynamics in single cells. Flow cytometry and mass spectrometry will be used to study networks of interacting proteins in colon tumor cells.

Class Format: two afternoons of lab, with one hour of lecture, per week

Requirements/Evaluation: lab participation, several short homework assignments, one lab report, a programming project, and a grant proposal

Prerequisites: BIOL 202; students who have not taken BIOL 202 but have taken BIOL 101 and CSCI 315 or PHYS 315, may enroll with permission of instructor. No prior computer programming experience is required.

Enrollment Limit: 12

Enrollment Preferences: seniors, then juniors, then sophomores

Expected Class Size: 12

Department Notes: does not satisfy the distribution requirement in the Biology major

Distributions:  (D3)  (QFR)

Attributes:  BGNP Core Courses;  BIMO Interdepartmental Electives;

Not offered current academic year

MATH 321 (S)  Knot Theory  (QFR)

Take a piece of string, tie a knot in it, and glue the ends together. The result is a knotted circle, known as a knot. For the last 100 years, mathematicians have studied knots, asking such questions as, “Given a nasty tangled knot, how do you tell if it can be untangled without cutting it open?” Some of the most interesting advances in knot theory have occurred in the last ten years. This course is an introduction to the theory of knots. Among other topics, we will cover methods of knot tabulation, surfaces applied to knots, polynomials associated to knots, and relationships between knot theory and chemistry and physics. In addition to learning the theory, we will look at open problems in the field.

Class Format: lecture

Requirements/Evaluation: evaluation will be based on problem sets, midterms, a paper and a final exam

Prerequisites: MATH 250 or permission of instructor

Enrollment Limit: 30

Expected Class Size: 25

Distributions:  (D3)  (QFR)

Spring 2019

LEC Section: 01    MWF 8:30 am - 9:45 am    Colin C. Adams

MATH 323 (S)  Applied Topology  (QFR)

In topology, one studies properties of an object that are preserved under rubber-like deformations, where one is allowed to twist and pull, but one cannot tear or glue. Hence a sphere is considered the same as a cube, but distinct from the surface of a doughnut. In recent years, topology has found applications in chemistry (knotted DNA molecules), economics (stability theory), Geographic Information Systems, cosmology (the shape of the
Universe), medicine (heart failure), robotics and electric circuit design, just to name some of the fields that have been impacted. In this course, we will learn the basics of topology, including point-set topology, geometric topology and algebraic topology, but all with the purpose of applying the theory to a broad array of fields.

**Class Format:** lecture

**Requirements/Evaluation:** evaluation will be based primarily on problem sets and exams

**Prerequisites:** MATH 250 or permission of instructor

**Enrollment Limit:** 40

**Enrollment Preferences:** none

**Expected Class Size:** 25

**Distributions:** (D3) (QFR)

Not offered current academic year

**MATH 326 (S) Differential Geometry** (QFR)

Differential Geometry is the study of curvature. In turn, curvature is the heart of geometry. The goal of this course is to start the study of curvature, concentrating on the curvature of curves and of surfaces, leading to the deep Gauss-Bonnet Theorem, which links curvature with topology.

**Class Format:** lecture

**Requirements/Evaluation:** evaluation will be based primarily on problem sets, midterms and a final exam

**Prerequisites:** MATH 250

**Enrollment Limit:** none

**Expected Class Size:** 25

**Distributions:** (D3) (QFR)

Not offered current academic year

**MATH 328 (F) Combinatorics** (QFR)

Combinatorics is a branch of mathematics that focuses on enumerating, examining, and investigating the existence of discrete mathematical structures with certain properties. This course provides an introduction to the fundamental structures and techniques in combinatorics including enumerative methods, generating functions, partition theory, and the principle of inclusion and exclusion.

**Class Format:** lecture

**Requirements/Evaluation:** homework assignments, proof portfolio, individual and group projects

**Extra Info:** may not be taken on a pass/fail basis; not available for the fifth course option

**Prerequisites:** MATH 200 and MATH 250 or permission of the instructor

**Enrollment Limit:** 30

**Enrollment Preferences:** discretion of the instructor

**Expected Class Size:** 30

**Distributions:** (D3) (QFR)

Not offered current academic year

**MATH 329 (S) Discrete Geometry** (QFR)

Discrete geometry is one of the oldest and most consistently vibrant areas of mathematics, stretching from the Platonic Solids of the ancient Greeks to the modern day applications of convex optimization and linear programming. In this tutorial we will learn about polygons and their higher-dimensional cousins, polyhedra and polytopes, and the various ways to describe, compute, and classify such objects. We will learn how these objects and ideas can be applied to other areas, from computation and optimization to studying areas of math like algebraic geometry. Throughout this course we will be engaging with mathematical work and literature from as old as 500 BCE and as recent as "posted to the internet yesterday."

**Class Format:** tutorial

**Requirements/Evaluation:** based primarily on participation, problem sets, oral presentations, an oral exam, and a final project

**Extra Info:** may not be taken on a pass/fail basis; not available for the fifth course option
**Prerequisites:** MATH 250 or permission of instructor

**Enrollment Limit:** 10

**Enrollment Preferences:** first-years and sophomores

**Expected Class Size:** 10

**Distributions:** (D3) (QFR)

Not offered current academic year

**MATH 331 (F) The little Questions (QFR)**

Using math competitions such as the Putnam Exam as a springboard, in this class we follow the dictum of the Ross Program and "think deeply of simple things". The two main goals of this course are to prepare students for competitive math competitions, and to get a sense of the mathematical landscape encompassing elementary number theory, combinatorics, graph theory, and group theory (among others). While elementary frequently is not synonymous with easy, we will see many beautiful proofs and "a-ha" moments in the course of our investigations. Students will be encouraged to explore these topics at levels compatible with their backgrounds.

**Class Format:** lecture

**Requirements/Evaluation:** evaluation will be based primarily on homework, classwork, and exams

**Prerequisites:** MATH 250 or permission of instructor

**Enrollment Limit:** 30

**Enrollment Preferences:** members or alternates of the Putnam team, Mathematics, Physics or Computer Science majors

**Expected Class Size:** 25

**Department Notes:** http://web.williams.edu/Mathematics/sjmiller/public_html/331/

**Distributions:** (D3) (QFR)

**Fall 2018**

LEC Section: 01 MWF 10:00 am - 10:50 am Steven J. Miller

**MATH 334 (S) Graph Theory (QFR)**

A graph is a collection of vertices, joined together by edges. In this course, we will study the sorts of structures that can be encoded in graphs, along with the properties of those graphs. We’ll learn about such classes of graphs as multi-partite, planar, and perfect graphs, and will see applications to such optimization problems as minimum colorings of graphs, maximum matchings in graphs, and network flows.

**Class Format:** lecture

**Requirements/Evaluation:** evaluation will be based primarily on problem sets and exams

**Prerequisites:** MATH 200 or MATH 250

**Enrollment Limit:** 35

**Enrollment Preferences:** Math majors

**Expected Class Size:** 20

**Distributions:** (D3) (QFR)

**Spring 2019**

LEC Section: 01 TR 9:55 am - 11:10 am Ralph E. Morrison

**MATH 337 (F) Electricity and Magnetism for Mathematicians (QFR)**

Maxwell's equations are four simple formulas, linking electricity and magnetism, that are among the most profound equations ever discovered. These equations led to the prediction of radio waves, to the realization that a description of light is also contained in these equations and to the discovery of the special theory of relativity. In fact, almost all current descriptions of the fundamental laws of the universe are deep generalizations of Maxwell's equations. Perhaps even more surprising is that these equations and their generalizations have led to some of the most important mathematical discoveries (where there is no obvious physics) of the last 25 years. For example, much of the math world was shocked at how these physics
generalizations became one of the main tools in geometry from the 1980s until today. It seems that the mathematics behind Maxwell is endless. This will be an introduction to Maxwell’s equations, from the perspective of a mathematician.

**Class Format:** lecture

**Requirements/Evaluation:** evaluation will be based primarily on performance on homework and exams

**Prerequisites:** MATH 250; no physics background required

**Enrollment Limit:** none

**Expected Class Size:** 15

**Distributions:** (D3) (QFR)

*Not offered current academic year*

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**MATH 341 (F) Probability** (QFR)

Crosslistings: STAT341 / MATH341

**Primary Crosslisting**

While probability began with a study of games, it has grown to become a discipline with numerous applications throughout mathematics and the sciences. Drawing on gaming examples for motivation, this course will present axiomatic and mathematical aspects of probability. Included will be discussions of random variables, expectation, independence, laws of large numbers, and the Central Limit Theorem. Many interesting and important applications will also be presented, potentially including some from coding theory, number theory and nuclear physics.

**Class Format:** lecture

**Requirements/Evaluation:** evaluation will be based primarily on homework, classwork, and exams

**Prerequisites:** MATH 250 or permission of the instructor

**Enrollment Limit:** 40

**Expected Class Size:** 20

**Distributions:** (D3) (QFR)

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**MATH 350 (F) Real Analysis** (QFR)

Real analysis is the theory behind calculus. It is based on a precise understanding of the real numbers, elementary topology, and limits. Topologically, nice sets are either closed (contain their limit points) or open (complement closed). You also need limits to define continuity, derivatives, integrals, and to understand sequences of functions.

**Class Format:** lecture/discussion

**Requirements/Evaluation:** evaluation will be based on homework, classwork, and exams

**Prerequisites:** MATH 150 or MATH 151 and MATH 250, or permission of instructor

**Enrollment Limit:** 40

**Expected Class Size:** 30

**Distributions:** (D3) (QFR)

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Fall 2018

LEC Section: 01  MWF 11:00 am - 11:50 am  Steven J. Miller

Spring 2019

LEC Section: 01  MWF 9:00 am - 9:50 am  Thomas A. Garrity

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**MATH 350 (F) Real Analysis** (QFR)

Real analysis is the theory behind calculus. It is based on a precise understanding of the real numbers, elementary topology, and limits. Topologically, nice sets are either closed (contain their limit points) or open (complement closed). You also need limits to define continuity, derivatives, integrals, and to understand sequences of functions.

**Class Format:** lecture/discussion

**Requirements/Evaluation:** evaluation will be based on homework, classwork, and exams

**Prerequisites:** MATH 150 or MATH 151 and MATH 250, or permission of instructor

**Enrollment Limit:** 40

**Expected Class Size:** 30

**Distributions:** (D3) (QFR)

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Fall 2018

LEC Section: 01  TF 2:35 pm - 3:50 pm  Leo Goldmakher

Spring 2019

LEC Section: 01  MWF 10:00 am - 10:50 am  Cesar E. Silva
MATH 351 (F)  Applied Real Analysis  (QFR)
Real analysis or the theory of calculus--derivatives, integrals, continuity, convergence--starts with a deeper understanding of real numbers, limits, and some topology. Applications of Real Analysis involve questions of existence and uniqueness of solutions, implicit definition of functions, infinite dimensional function spaces, and tools from calculus of variations to construct optimal controls and minimizing curves and surfaces.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on exams, homework and quizzes
Prerequisites: MATH 150 and MATH 250, or permission of instructor
Enrollment Limit: 50
Expected Class Size: 20
Distributions: (D3) (QFR)

Fall 2018
LEC Section: 01  MWF 9:00 am - 9:50 am  Stewart D. Johnson

MATH 355 (F)  Abstract Algebra  (QFR)
Algebra gives us tools to solve equations. The integers, the rationals, and the real numbers have special properties which make algebra work according to the circumstances. In this course, we generalize algebraic processes and the sets upon which they operate in order to better understand, theoretically, when equations can and cannot be solved. We define and study abstract algebraic structures such as groups, rings, and fields, as well as the concepts of factor group, quotient ring, homomorphism, isomorphism, and various types of field extensions. This course introduces students to abstract rigorous mathematics.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on problem sets and exams
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 30
Expected Class Size: 25
Distributions: (D3) (QFR)

Fall 2018
LEC Section: 01  MWF 10:00 am - 10:50 am  Susan R. Loepp

Spring 2019
LEC Section: 01  TR 9:55 am - 11:10 am  Allison Pacelli
LEC Section: 02  TR 11:20 am - 12:35 pm  Allison Pacelli

MATH 361 (F)  Theory of Computation  (QFR)
Crosslistings: CSCI361 / MATH361
Secondary Crosslisting
This course introduces a formal framework for investigating both the computability and complexity of problems. We study several models of computation including finite automata, regular languages, context-free grammars, and Turing machines. These models provide a mathematical basis for the study of computability theory--the examination of what problems can be solved and what problems cannot be solved--and the study of complexity theory--the examination of how efficiently problems can be solved. Topics include the halting problem and the P versus NP problem.

Class Format: lecture
Requirements/Evaluation: evaluation will be based on problem sets, a midterm examination, and a final examination
Prerequisites: CSCI 256 or both a 300-level MATH course and permission of instructor
Enrollment Limit: 34
Enrollment Preferences: current or expected Computer Science majors
Expected Class Size: 34
MATH 367 (S) Homological Algebra (QFR)
Though a relatively young subfield of mathematics, Homological Algebra has earned its place by supplying powerful tools to solve questions in the much older fields of Commutative Algebra, Algebraic Geometry and Representation Theory. This class will introduce theorems and tools of Homological Algebra, grounding its results in applications to polynomial rings and their quotients. We will focus on some early groundbreaking results and learn some of Homological Algebra’s most-used constructions. Possible topics include tensor products, chain complexes, homology, Ext, Tor and Hilbert’s Syzygy Theorem.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on homework and exams
Extra Info: may not be taken on a pass/fail basis
Prerequisites: MATH 355
Enrollment Limit: 20
Enrollment Preferences: junior and senior math majors
Expected Class Size: 12
Distributions: (D3) (QFR)
Not offered current academic year

MATH 368 (S) Positive Characteristic Commutative Algebra (QFR)
In commutative algebra, one of the most basic invariants of a ring is its characteristic. This is the smallest multiple of 1 that equals 0. Working over a ring of characteristic zero, versus a ring of characteristic p>0, dramatically changes the proof techniques available to us. This realization has had tremendous consequences in commutative algebra. One of the most useful tools in characteristic p is the Frobenius homomorphism. In this course we will study several standard notions in commutative algebra, such as regularity of a ring, Cohen-Macaulayness, and being normal and we will see how various “splittings” of the Frobenius allow us to easily detect these properties. Many of these methods are not only applicable to commutative algebra, but also to number theory and algebraic geometry.

Class Format: lecture
Requirements/Evaluation: homework and a final exam
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 355 or permission of instructor
Enrollment Limit: 30
Enrollment Preferences: Math majors primarily, and juniors and seniors secondarily
Expected Class Size: 15
Distributions: (D3) (QFR)
Not offered current academic year

MATH 372 (F) Complex Analysis (QFR)
The calculus of complex-valued functions turns out to have unexpected simplicity and power. As an example of simplicity, every complex-differentiable function is automatically infinitely differentiable. As examples of power, the so-called “residue calculus” permits the computation of “impossible” integrals, and “conformal mapping” reduces physical problems on very general domains to problems on the round disc. The easiest proof of the Fundamental Theorem of Algebra, not to mention the first proof of the Prime Number Theorem, used complex analysis. We will discuss these and other topics as time permits (such as the Riemann Mapping Theorem, Special Functions, and the Central Limit Theorem).

Class Format: lecture
MATH 374 (F)  Topology  (QFR)

Topology is the study of when one geometric object can be continuously deformed and twisted into another object. Determining when two objects are topologically the same is incredibly difficult and is still the subject of a tremendous amount of research, including recent work on the Poincaré Conjecture, one of the million-dollar millennium-prize problems. The main part of the course on point-set topology establishes a framework based on "open sets" for studying continuity and compactness in very general spaces. The second part on homotopy theory develops refined methods for determining when objects are the same. We will prove for example that you cannot twist a basketball into a doughnut.

Class Format: lecture

Requirements/Evaluation: homework, tutorials, and exams

Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option

Prerequisites: MATH 350 or 351; not open to students who have taken MATH 323

Enrollment Limit: 30

Expected Class Size: 10

Distributions: (D3) (QFR)

Not offered current academic year
MATH 379 (F) Asymptotic Analysis in Differential Equations

Asymptotic Analysis is a fascinating subfield of differential equations in which interesting and unexpected phenomena can occur. Roughly speaking, the problem is this: Given a differential equation depending on a parameter epsilon, what happens to the solutions to the equation as we let epsilon go to 0? After an extensive survey of examples, we will cover asymptotic evaluation of integrals, such as stationary phase and Laplace's method, multiple scales, WKB approximations, averaging methods, matched asymptotic expansions, and boundary layers. If time permits, we will also discuss bifurcation theory and the Nash-Moser Inverse Function Theorem.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on homework and exams
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 350 or MATH 351
Enrollment Limit: 25
Expected Class Size: 15
Distributions: (D3)
Not offered current academic year

MATH 382 (S) Harmonic Analysis (QFR)

Harmonic Analysis is a diverse field which includes Fourier Analysis, one of the major tools of modern mathematics. Applications range from mathematical topics such as partial differential equations and number theory to more applied ones such as signal processing and medical imaging. The course will begin with an introduction to the Fourier Transform and will cover a wide variety of topics including singular integral operators, maximal operators and wavelets as the semester progresses. Along the way applications from partial differential equations and ergodic theory will arise with a highlight being the almost everywhere convergence of Fourier series.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on exams, homework, quizzes and a project
Prerequisites: MATH 350 or MATH 351 or permission of the instructor
Enrollment Limit: 30
Enrollment Preferences: lottery
Expected Class Size: 15
Distributions: (D3) (QFR)
Not offered current academic year

MATH 392 (F) Undergraduate Research Topics in Graph Theory (QFR)

Graph theory is a vibrant area of research with many applications to the social sciences, psychology, and economics. In this tutorial we focus on two topics of mathematical research in graph theory: evasion-pursuit games on graphs and domination theory. Students in this project-based tutorial will select among the presented topics, and will begin original research on an open problem in the field. Student assessment is based on problem sets, drafts of research project manuscript, and a final oral class presentation.

Class Format: tutorial
Requirements/Evaluation: homework assignments, oral presentations, and written project manuscript
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 355 or permission of the instructor
Enrollment Limit: 10
Enrollment Preferences: programming experience, students with interests in the intersection of combinatorics and graph theory
Expected Class Size: 10
Distributions: (D3) (QFR)
Not offered current academic year
MATH 394 (S)  Galois Theory  (QFR)
Some equations--such as $x^5 - 1 = 0$--are easy to solve. Others--such as $x^5 - x - 1 = 0$--are very hard, if not impossible (using standard mathematical operations). Galois discovered a deep connection between field theory and group theory that led to a criterion for checking whether or not a given polynomial can be easily solved. His discovery also led to many other breakthroughs, for example proving the impossibility of squaring the circle or trisecting a typical angle using compass and straightedge. From these not-so-humble beginnings, Galois theory has become a fundamental concept in modern mathematics, from topology to number theory. In this course we will develop the theory and explore its applications to other areas of math.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on written homeworks, oral presentations, and exams
Extra Info: may not be taken on a pass/fail basis
Prerequisites: MATH 355
Enrollment Limit: 15
Enrollment Preferences: discretion of the instructor
Expected Class Size: 10
Distributions: (D3) (QFR)
Not offered current academic year

MATH 397 (F)  Independent Study: Mathematics
Directed 300-level independent study in Mathematics.
Class Format: independent study
Prerequisites: permission of department
Distributions: (D3)

Fall 2018
IND Section: 01  TBA  Susan R. Loepp
IND Section: 02  TBA  Steven J. Miller

MATH 398 (S)  Independent Study: Mathematics
Directed 300-level independent study in Mathematics.
Class Format: independent study
Prerequisites: permission of department
Distributions: (D3)

Spring 2019
IND Section: 01  TBA  Susan R. Loepp

MATH 401 (F)  Functional Analysis  (QFR)
Functional analysis can be viewed as linear algebra on infinite-dimensional spaces. It is a central topic in Mathematics, which brings together and extends ideas from analysis, algebra, and geometry. Functional analysis also provides the rigorous mathematical background for several areas of theoretical physics (especially quantum mechanics). We will introduce infinite-dimensional spaces (Banach and Hilbert spaces) and study their properties. These spaces are often spaces of functions (for example, the space of square-integrable functions). We will consider linear operators on Hilbert spaces and investigate their spectral properties. A special attention will be dedicated to various operators arising from mathematical physics, especially the Schrodinger operator.

Class Format: lecture
Requirements/Evaluation: weekly problem sets, two midterm exams, final exam
Prerequisites: MATH 350 or 351 or permission of instructor
Enrollment Limit: 40
**Enrollment Preferences:** Mathematics and Physics majors; seniors

**Expected Class Size:** 30

**Distributions:** (D3) (QFR)

Not offered current academic year

**MATH 402 (S) Measure Theory and Probability** (QFR)

The study of measure theory arose from the study of stochastic (probabilistic) systems. Applications of measure theory lie in biology, chemistry, physics as well as in economics. In this course, we develop the abstract concepts of measure theory and ground them in probability spaces. Included will be Lebesgue and Borel measures, measurable functions (random variables). Lebesgue integration, distributions, independence, convergence and limit theorems. This material provides good preparation for graduate studies in mathematics, statistics and economics.

**Class Format:** lecture/discussion

**Requirements/Evaluation:** evaluation will be based primarily on performance on homework assignments and exams

**Prerequisites:** MATH 350 or MATH 351 or permission of instructor

**Enrollment Limit:** 40

**Expected Class Size:** 30

**Distributions:** (D3) (QFR)

Spring 2019

LEC Section: 01      Cancelled

**MATH 403 (S) Measure and Ergodic Theory** (QFR)

An introduction to measure theory and ergodic theory. Measure theory is a generalization of the notion of length and area, has been used in the study of stochastic (probabilistic) systems. The course covers the construction of Lebesque and Borel measures, measurable functions, and Lebesque integration. Ergodic theory studies the probabilistic behavior of dynamical systems as they evolve through time, and is based on measure theory. The course will cover basic notions, such as ergodic transformations, weak mixing, mixing, and Bernoulli transformations, and transformations admitting and not admitting an invariant measure. There will be an emphasis on specific examples such as group rotations, the binary odometer transformations, and rank-one constructions. The Ergodic Theorem will also be covered, and will be used to illustrate notions and theorems from measure theory.

**Class Format:** lecture

**Requirements/Evaluation:** homework and exams

**Prerequisites:** MATH 350 or MATH 351 or permission of instructor

**Enrollment Limit:** 25

**Enrollment Preferences:** Mathematics majors

**Expected Class Size:** 15-20

**Department Notes:** senior major course

**Distributions:** (D3) (QFR)

Spring 2019

LEC Section: 01    MWF 12:00 pm - 12:50 pm     Cesar E. Silva

**MATH 407 (F) Dance of the Primes** (QFR)

Prime numbers are the building blocks for all numbers and hence for most of mathematics. Though there are an infinite number of them, how they are spread out among the integers is still quite a mystery. Even more mysterious and surprising is that the current tools for investigating prime numbers involve the study of infinite series. Function theory tells us about the primes. We will be studying one of the most amazing functions known: the Riemann Zeta Function. Finding where this function is equal to zero is the Riemann Hypothesis and is one of the great, if not greatest, open problems in mathematics. Somehow where these zeros occur is linked to the distribution of primes. We will be concerned with why anyone would care about this conjecture. More crassly, why should solving the Riemann Hypothesis be worth one million dollars? (Which is what you will get if you solve it, beyond
Using mathematics to study natural phenomena has become ubiquitous over the past couple of decades. In this tutorial, we will study mathematical models comprised of both deterministic and stochastic differential equations that are developed to understand ecological dynamics and, in many cases, evaluate the dynamical consequences of policy decisions. We will learn how to understand these models through both standard analytic techniques such as stability and bifurcation analysis as well as through simulation using computer programs such as MATLAB. Possible topics include fisheries management, disease ecology, control of invasive species, and predicting critical transitions in ecological systems.

Class Format: tutorial
Requirements/Evaluation: written and programming assignments, oral presentations, and exams
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 250 or permission of instructor; Math 209 preferred
Enrollment Limit: 10
Enrollment Preferences: programming experience, students with interests in the intersection of math and biology
Expected Class Size: 10
Department Notes: Does not satisfy the distribution requirement in the Biology major
Distributions: (D3) (QFR)
Attributes: ENVI Natural World Electives; EVST Methods Courses; PHLH Methods in Public Health;
Not offered current academic year
We all know that integers can be factored into prime numbers and that this factorization is essentially unique. In more general settings, it often still makes sense to factor numbers into "primes," but the factorization is not necessarily unique! This surprising fact was the downfall of Lamé's attempted proof of Fermat's Last Theorem in 1847. Although a valid proof was not discovered until over 150 years later, this error gave rise to a new branch of mathematics: algebraic number theory. In this course, we will study factorization and other number-theoretic notions in more abstract algebraic settings, and we will see a beautiful interplay between groups, rings, and fields.

**Class Format:** lecture/seminar

**Requirements/Evaluation:** evaluation will be based primarily on homework assignments and exams

**Prerequisites:** MATH 355, or permission of instructor

**Enrollment Limit:** 40

**Expected Class Size:** 20

**Distributions:** (D3) (QFR)

Not offered current academic year

How many primes are smaller than x? How many divisors does an integer n have? How many different numbers appear in the N x N multiplication table? Over the course of the past 150 years, tremendous progress has been made towards resolving these and similar questions in number theory, relying on tools and methods from analysis. The goal of this tutorial is to explain and motivate the ubiquitous appearance of analysis in modern number theory—a surprising fact, given that analysis is concerned with continuous functions, while number theory is concerned with discrete objects (integers, primes, divisors, etc). Topics to be covered include: asymptotic analysis, partial and Euler-Maclaurin summation, counting divisors and Dirichlet's hyperbola method, the randomness of prime factorization and the Erdos-Kac theorem, the partition function and the saddle point method, the prime number theorem and the Riemann zeta function, primes in arithmetic progressions and Dirichlet L-functions, the Goldbach conjecture and the circle method, gaps between primes, and other topics as time and interest allow.

**Class Format:** tutorial

**Requirements/Evaluation:** tutorial format (problem sets and presentations)

**Extra Info:** may not be taken on a pass/fail basis; not available for the fifth course option

**Prerequisites:** MATH 350 or MATH 351, MATH 372 (may be taken concurrently), familiarity with modular arithmetic

**Enrollment Limit:** 10

**Enrollment Preferences:** Mathematics majors

**Expected Class Size:** 10

**Distributions:** (D3) (QFR)

Not offered current academic year

A quandle is an algebraic object that, like a group, has a "multiplication" of pairs of elements that satisfies certain axioms. But the quandle axioms are very different from the group axioms, and quandles turn out to be incredibly useful when considering the mathematical theory of knots. In this course, we will learn about this relatively new area of research (1982) and learn some knot theory and see how quandles apply to both classical knot theory and the relatively new area of virtual knot theory (1999).

**Class Format:** lecture

**Requirements/Evaluation:** problem sets, tests, and a 3-page paper

**Extra Info:** may not be taken on a pass/fail basis; not available for the fifth course option

**Prerequisites:** MATH 355

**Enrollment Limit:** 40
Enrollment Preferences: discretion of the instructor
Expected Class Size: 15
Distributions: (D3) (QFR)

Not offered current academic year

MATH 422 (F) Algebraic Topology  (QFR)
Is a sphere really different from a torus? Can a sphere be continuously deformed to a point? Algebraic Topology concerns itself with the classification and study of topological spaces via algebraic methods. The key question is this: How do we really know when two spaces are different and in what senses can we claim they are the same? Our answer will use several algebraic tools such as groups and their normal subgroups. In this course we will develop several notions of "equality" starting with the existence of homeomorphisms between spaces. We will then explore several weakenings of this notion, such as homotopy equivalence, having isomorphic homology or fundamental groups, and having homeomorphic universal covers.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on homework and exams
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 355 or permission of instructor
Enrollment Limit: 30
Enrollment Preferences: Math majors primarily, and Juniors and Seniors secondarily
Expected Class Size: 15
Distributions: (D3) (QFR)

Not offered current academic year

MATH 424 (F) Geometry, Surfaces and Billiards
Mathematical billiards is the study of a ball bouncing around in a table--a rectangle in the popular pub game, but any shape of table for us, including triangles and ellipses. The geometry of billiards is elegant, and is related to surfaces, fractals, and even continued fractions. We will study many types of billiards and surfaces, and take time to explore some beautiful examples and ideas.

Class Format: lecture
Requirements/Evaluation: based on work in class, problem sets, an exam and a project.
Extra Info: not available for the fifth course option
Prerequisites: MATH 350/351 and MATH 355
Enrollment Limit: 25
Expected Class Size: 15
Distributions: (D3)

Not offered current academic year

MATH 431 (F) Nonlinear Waves, Solitons  (QFR)
Waves arise in scientific and engineering disciplines such as acoustics, optics, fluid/solid mechanics, electromagnetism and quantum mechanics. Although linear waves are well understood, the study of nonlinear wave phenomena remains an active field of research and a source of inspiration and challenge for several areas of mathematics. We discuss traveling waves, shallow water models, wave steepening, solitons and blowup. Additional topics may include shocks, weak solutions and conservation laws.

Class Format: lecture
Requirements/Evaluation: problem sets, exams, and final project
Prerequisites: MATH 209/210 and MATH 350/351, or permission of the instructor
Enrollment Limit: 40
Expected Class Size: 15
Distributions: (D3) (QFR)

Not offered current academic year
MATH 433 (F) Mathematical Modeling (QFR)

Mathematical modeling means (1) translating a real-life problem into a mathematical object, and (2) studying that object using mathematical techniques, and (3) interpreting the results in order to learn something about the real-life problem. Mathematical modeling is used in biology, economics, chemistry, geology, sociology, political science, art, and countless other fields. This is an advanced, seminar-style, course appropriate for students who have a strong enthusiasm for applied mathematics.

Class Format: discussion, research

Requirements/Evaluation: writing assignments, modeling activities, presentations, research project

Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option

Prerequisites: MATH 250, MATH 309 or similar, and some experience with computer programming (equivalent to CSCI 134 or MATH 307)

Enrollment Limit: 24

Enrollment Preferences: Professor's discretion

Expected Class Size: 20

Distributions: (D3) (QFR)

Fall 2018

LEC Section: 01    TR 11:20 am - 12:35 pm     Chad M. Topaz

MATH 434 (S) Applied Dynamics and Optimal Control (QFR)

We seek to understand how dynamical systems evolve, how that evolution depends on the various parameters of the system, and how we might manipulate those parameters to optimize an outcome. We will explore the language of dynamics by deepening our understanding of differential and difference equations, study parameter dependence and bifurcations, and explore optimal control through Pontryagin's maximum principle and Hamilton-Jacobi-Bellman equations. These tools have broad application in ecology, economics, finance, and engineering, and we will draw on basic models from these fields to motivate our study.

Class Format: lecture

Requirements/Evaluation: evaluation will be based primarily on exams and homework assignments

Prerequisites: MATH 209 or PHYS 210, and MATH 350 or 351, or permission of the instructor

Enrollment Limit: 40

Expected Class Size: 25

Distributions: (D3) (QFR)

Not offered current academic year

MATH 453 (F) Partial Differential Equations (QFR)

Partial differential equations (PDE) arise as mathematical models of phenomena in chemistry, ecology, economics, electromagnetics, fluid dynamics, neuroscience, thermodynamics, and more. We introduce PDE models and develop techniques for studying them. Topics include: derivation, classification, and physical interpretation of canonical PDE; solution techniques, including separation of variables, series solutions, integral transforms, and characteristics; and application to problems in the natural and social sciences.

Class Format: lecture

Requirements/Evaluation: quizzes/exams, problem sets, projects and activities

Extra Info: may not be taken on a pass/fail basis

Prerequisites: MATH 150-151; MATH 209 or MATH/PHYS 210 or MATH 309

Enrollment Limit: 30

Enrollment Preferences: Professor's discretion

Expected Class Size: 25

Distributions: (D3) (QFR)

Not offered current academic year
MATH 456 (F) Representation Theory  (QFR)
Representation theory has applications in fields such as physics (via models for elementary particles), engineering (considering symmetries of structures), and even in voting theory (voting for committees in agreeable societies). This course will introduce the concepts and techniques of the representation theory of finite groups, and will focus on the representation theory of the symmetric group. We will undertake this study through a variety of perspectives, including general representation theory, combinatorial algorithms, and symmetric functions.

Class Format: lecture
Requirements/Evaluation: evaluation will be based primarily on homework, in class presentations, and exams
Extra Info: may not be taken on a pass/fail basis
Prerequisites: MATH 355
Enrollment Limit: 40
Enrollment Preferences: junior and senior Math majors
Expected Class Size: 15
Distributions: (D3) (QFR)
Not offered current academic year

MATH 458 (S) Algebraic Combinatorics  (QFR)
Algebraic combinatorics is a branch of mathematics at the intersection of combinatorics and algebra. On the one hand, we study combinatorial structures using algebraic techniques, while on the other we use combinatorial arguments and methods to solve problems in algebra. This course will focus on the study of symmetric functions, young tableaux, matroids, graph theory, and other related topics.

Class Format: seminar
Requirements/Evaluation: homework assignments, proof portfolio, individual and group projects
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 200 and MATH 355
Enrollment Limit: 25
Enrollment Preferences: seniors
Expected Class Size: 20
Distributions: (D3) (QFR)
Distribution Notes: QFR: Mathematics course in the area of algebraic combinatorics

Spring 2019
SEM Section: 01    TR 8:30 am - 9:45 am     Pamela E. Harris

MATH 459 (S) Applied Partial Differential Equations  (QFR)
Partial differential equations (PDE) arise as mathematical models of phenomena in chemistry, ecology, economics, electromagnetics, epidemiology, fluid dynamics, neuroscience, and much more. Furthermore, the study of partial differential equations connects with diverse branches of mathematics including analysis, geometry, algebra, and computation. Adopting an applied viewpoint, we develop techniques for studying PDE. We draw from a body of knowledge spanning classic work from the time of Isaac Newton right up to today's cutting edge applied mathematics research. This tutorial is appropriate as a second course in differential equations. In this tutorial, students will: build and utilize PDE-based models; determine the most appropriate tools to apply to a PDE; apply the aforementioned tools; be comfortable with open-ended scientific work; read applied mathematical literature; communicate applied mathematics clearly, precisely, and appropriately; collaborate effectively.

Class Format: tutorial
Requirements/Evaluation: participation, problem sets, oral presentations, oral exams, and a final project
Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option
Prerequisites: MATH 209 or MATH/PHYS 210 or MATH 309 or permission of instructor
Enrollment Limit: 10
Enrollment Preferences: students with an interest in applied mathematics, selected to create a diverse set of tutorial participants

Expected Class Size: 10

Department Notes: students who have taken MATH 453 may not enroll in MATH 458T without permission of the instructor

Distributions: (D3) (QFR)

Distribution Notes: QFR: This tutorial involves regular and substantial problem sets in which quantitative/formal reasoning skills are practiced and evaluated.

Spring 2019
TUT Section: T1   TBA   Chad M. Topaz

MATH 466 (F) Advanced Applied Analysis (QFR)

This course further develops and explores topics and concepts from real analysis, with special emphasis on introducing students to subject matter and techniques that are useful for graduate study in mathematics or an allied field, as well as applications in industry. Topics include Benford's law of digit bias, random matrix theory, and Fourier analysis, and as time permits additional areas based on student interest from analytic number theory, generating functions and probabilistic methods. This will be an intense, fast paced class which will give a flavor for graduate school. In addition to standard homework problems, students will assist in writing both reviews for MathSciNet and referee reports for papers for journals, write programs to investigate and conjecture, and read classic and current research papers, and possibly apply these and related methods to real world problems.

Class Format: lecture

Requirements/Evaluation: homework, exams, possible paper/presentation

Prerequisites: MATH 350 or MATH 351

Enrollment Limit: 40

Enrollment Preferences: discretion of the instructor

Expected Class Size: 30

Distributions: (D3) (QFR)

Not offered current academic year

MATH 474 (S) Tropical Geometry (QFR)

This course offers an introduction to tropical geometry, a young subject that has already established deep connections between itself and pure and applied mathematics. We will study a rich variety of objects arising from polynomials over the min-plus semiring, where addition is defined as taking a minimum, and multiplication is defined as usual addition. We will learn how these polyhedral objects connect to other areas of mathematics like algebraic geometry, and how they can be applied to solve problems in scheduling theory, phylogenetics, and other diverse fields.

Class Format: lecture

Requirements/Evaluation: based primarily on participation, problem sets, quizzes, exams, and a final project

Extra Info: may not be taken on a pass/fail basis; not available for the fifth course option

Prerequisites: MATH 355 or permission of instructor

Enrollment Limit: 25

Enrollment Preferences: senior Math majors

Expected Class Size: 15

Distributions: (D3) (QFR)

Not offered current academic year

MATH 475 (S) Methods in Mathematical Fluid Dynamics (QFR)

Crosslistings: MATH475 / PHYS475

Primary Crosslisting

The mathematical study of fluids is an exciting field with applications in areas such as engineering, physics and biology. The applied nature of the subject has led to important developments in aerodynamics and hydrodynamics. From ocean currents and exploding supernovae to weather prediction
and even traffic flow, several partial differential equations (pde) have been proposed as models to study fluid phenomena. This course is designed to both, introduce students to some of the techniques used in mathematical fluid dynamics and lay down a foundation for future research in this and other related areas. Briefly, we start with the method of characteristics, a useful tool in the study of pde. Symmetry and geometrical arguments, special solutions, energy methods, particle trajectories, and techniques from ordinary differential equations (ode) are also discussed. A special focus will be on models from hydrodynamics. These include the KdV and the Camassa Holm equations (and generalizations thereof), and the Euler equations of ideal fluids. Mainly, we will be concerned with models whose solutions depend on time and one spatial variable, although depending on student interest and time, we may also investigate higher-dimensional models.

Class Format: lecture
Requirements/Evaluation: problem sets and final project
Extra Info: may not be taken on a pass/fail basis
Prerequisites: MATH 151, MATH 250, and MATH 350 or 351; some background in pde/ode would be helpful but not required
Enrollment Limit: 40
Enrollment Preferences: senior Mathematics majors
Expected Class Size: 25
Distributions: (D3) (QFR)
Not offered current academic year

MATH 478 (S) On Expressing Numbers (QFR)
The real numbers are overall mysterious. Attempts even to describe different real numbers can quickly lead to deep, open questions in mathematics. For example, writing numbers via their decimal expansions leads to the result that a number is rational precisely when the decimal expansion is eventually periodic. There is an entirely different method for describing real numbers: continued fractions, which go back thousands of years. Here every real number can be captured by a sequence of integers (just like for the decimal expansion) but now eventually periodicity corresponds to the number being a square root. The mathematics of continued fractions, and especially their higher dimensional generalizations, lead to a great deal of mathematics. We will be using tools from linear algebra, functional analysis, dynamical systems, ergodic theory and algebraic number theory to explore the best way to express a real number.

Class Format: lecture
Requirements/Evaluation: exams and homework
Prerequisites: MATH 350 or MATH 351, and MATH 355
Enrollment Limit: none
Enrollment Preferences: Seniors
Expected Class Size: 15
Distributions: (D3) (QFR)
Not offered current academic year

MATH 487 (S) Computational Algebraic Geometry (QFR)
Algebraic geometry is the study of shapes described by polynomial equations. It has been a major part of mathematics for at least the past two hundred years, and has influenced a tremendous amount of modern mathematics, ranging from number theory to robotics. In this course, we will develop the Ideal-Variety Correspondence that ties geometric shapes to abstract algebra, and will use computational tools to explore this theory in a very explicit way.

Class Format: lecture
Requirements/Evaluation: evaluation will be based on homework, exams, and final project
Extra Info: may not be taken on a pass/fail basis
Prerequisites: MATH 355
Enrollment Limit: 40
Enrollment Preferences: instructor decision
Expected Class Size: 15
Department Notes: This course is not a senior seminar, and so it does not fulfill the senior seminar requirement for the math major
MATH 493 (F) Senior Honors Thesis: Mathematics
Mathematics senior honors thesis. Each student carries out an individual research project under the direction of a faculty member that culminates in a thesis. See description under *The Degree with Honors in Mathematics.*

**Class Format:** independent study

**Extra Info:** this is part of a full-year thesis (493-494)

**Distributions:** (D3)

MATH 494 (S) Senior Honors Thesis: Mathematics
Mathematics senior honors thesis. Each student carries out an individual research project under the direction of a faculty member that culminates in a thesis. See description under *The Degree with Honors in Mathematics.*

**Class Format:** independent study

**Extra Info:** this is part of a full-year thesis (493-494)

**Distributions:** (D3)

MATH 497 (F) Independent Study: Mathematics
Directed 400-level independent study in Mathematics.

**Class Format:** independent study

**Prerequisites:** permission of department

**Distributions:** (D3)

MATH 498 (S) Independent Study: Mathematics
Directed 400-level independent study in Mathematics.

**Class Format:** independent study

**Prerequisites:** permission of department

**Distributions:** (D3)

MATH 499 (F) Senior Colloquium
Mathematics senior colloquium. Meets every week for two hours both fall and spring. Senior majors must participate at least one hour a week. This colloquium is in addition to the regular four semester-courses taken by all students.

Class Format: colloquium

Distributions:

Fall 2018
LEC Section: 01   MW 1:00 pm - 1:45 pm   Susan R. Loepp

Spring 2019
LEC Section: 01   MW 1:00 pm - 1:45 pm   Susan R. Loepp