MATHEMATICS AND STATISTICS (Div III)

MATHEMATICS
Chair: Professor Mihai Stoiciu

- Colin C. Adams, Thomas T. Read Professor of Mathematics; on leave 2020-2021
- Julie C. Blackwood, Associate Professor of Mathematics; on leave Spring 2021
- Xizhen Cai, Assistant Professor of Statistics
- Josh Carlson, Visiting Assistant Professor of Mathematics
- Richard D. De Veaux, C. Carlisle and Margaret Tippit Professor of Statistics
- Thomas A. Garrity, Webster Atwell Class of 1921 Professor of Mathematics; on leave 2020-2021
- Eva G. Goedhart, Visiting Assistant Professor of Mathematics
- Leo Goldmakher, Associate Professor of Mathematics
- Pamela E. Harris, Associate Professor of Mathematics
- Stewart D. Johnson, Professor of Mathematics
- Bernhard Klingenberg, Professor of Statistics
- Haydee M. A. Lindo, Assistant Professor of Mathematics; on leave 2020-2021
- Susan R. Loepp, William R. Kenan, Jr. Professor of Mathematics
- Steven J. Miller, Professor of Mathematics
- Ralph E. Morrison, Assistant Professor of Mathematics
- Shaoyang Ning, Assistant Professor of Statistics
- Allison Pacelli, Professor of Mathematics
- Lori A. Pedersen, Lecturer in Mathematics
- Anna M. Plantinga, Assistant Professor of Statistics
- Cesar E. Silva, Hagey Family Professor of Mathematics; on leave 2020-2021
- Mihai Stoiciu, Chair and Professor of Mathematics
- Chad M. Topaz, Professor of Mathematics
- Laurie L. Tupper, Assistant Professor of Statistics
- Daniel B. Turek, Assistant Professor of Statistics
- Elizabeth M. Upton, Assistant Professor of Statistics; on leave Spring 2021
- John D. Wiltshire-Gordon, Visiting Assistant Professor of Mathematics

MAJOR IN MATHEMATICS

The major in Mathematics is designed to meet two goals: to introduce some of the central ideas in a variety of areas of mathematics and statistics, and to develop problem-solving ability by teaching students to combine creative thinking with rigorous reasoning. Mathematics is a gateway to many career paths including statistics, teaching, consulting, business, engineering, finance, actuarial studies and applied mathematics. Students are strongly encouraged to consult with the department faculty on choosing courses appropriate to an individualized program of study.

REQUIREMENTS (9 courses plus colloquium)

The major in Mathematics consists of nine courses taken at Williams plus the colloquium requirement. Mathematics is highly cumulative, and students should plan a route to completing the major that ensures the proper sequencing and prerequisites for all needed courses. Note that not all upper level courses are offered every year.
Calculus (2 courses)
  Mathematics 140 Calculus II
  Mathematics 150 or 151 Multivariable Calculus

Applied/Discrete Mathematics/Statistics (1 course)
  or Mathematics 210 Mathematical Methods for Scientists (Same as Physics 210)
  or Mathematics 200 Discrete Mathematics
  or Statistics 201 Statistics and Data Analysis
  Mathematics 309 Differential Equations
  or a more advanced applied/discrete/statistics course with prior department approval

Core Courses (3 courses)
  Mathematics 250 Linear Algebra
  Mathematics 350 Real Analysis or Mathematics 351 Applied Real Analysis
  Mathematics 355 Abstract Algebra

Completion (3 courses plus colloquium)
  Two mathematics or statistics electives from courses numbered 300 and above
  One Senior Seminar: Any mathematics or statistics course numbered between 400 and 479, taken in the junior or senior year.
  Participation in the Department Colloquium, in which all senior majors present a talk on a mathematical or statistical topic of their choice. Each major must attend at least 20 colloquia (reduced to 15 during the Academic Year 2020-2021), and up to 5 attendances may be counted in their junior year. Students engaged in study away may petition the department in advance to count up to 5 suitable colloquia attendances from their study away program.

Pass/Fail policies during the Academic Year 2020-2021
  Information about the Department of Mathematics and Statistics Pass/Fail policies during the Academic Year 2020-2021 can be found here.

ADVANCED PLACEMENT
  Students who come to Williams with advanced placement will be moved up in the Mathematics major, and should consult with faculty to be placed in the best class reflecting their experience and background. A student who places out of a course substitutes another course of equal or higher level in Mathematics or Statistics to complete the nine course major. Students should select courses best suited to their preparation and goals, and consult with the department faculty concerning appropriate courses and placement. The department reserves the right to refuse registration in any course for which the student is overqualified.
  For Example, a student starting in MATH 130 might take MATH 130 and 140 the first year, MATH 150 and MATH 200 the second year, MATH 250 and MATH 350 the third year, MATH 355 and a senior seminar the fourth year, plus the two required electives some time. Students are encouraged to consult freely with any math faculty about course selection and anything else.

CALCULUS PLACEMENT
Recommended placement for students who have taken an Advanced Placement Examination in Calculus (AB or BC) is:
  BC 1, 2 or AB 2, 3 Math 140
  AB 4 or 5 Math 150
  BC 3, 4 or 5 Math 151

Consult with department faculty for any Calculus or Statistics placement questions. Students who have had calculus in high school, whether or not they took the Advanced Placement Examination, are barred from 130 unless they obtain permission from the instructor.

NOTES
  Substitutions, Study Abroad, and Transfer Credit: In some cases, and with prior permission of the Mathematics and Statistics Department, appropriate courses from other institutions or a course from another Williams department may be substituted for electives. Programs like the “Budapest Semester in Mathematics” are recommended for majors who wish to focus on mathematics away. The department, though, normally accommodates students who select other study away programs. The department offers its core courses in both the fall and the spring to allow
students to spend more easily a semester away.

**Double Counting:** No course may count towards two different majors.

**Planning Courses:** Core courses Mathematics 350/351 and 355, are normally offered every year. Most other 300-level topics are offered in alternate years. Topology, Complex Analysis, and second courses in real analysis and abstract algebra are normally offered at least every other year.

Each 400-level topic is normally offered every two to four years. Students should check with the department before planning far into the future.

**Course Admission:** Courses are normally open to all students meeting the prerequisites, subject to any course caps. Students with questions about the level at which courses are conducted are invited to consult department faculty.

**FAQ**

Students MUST contact departments/programs BEFORE assuming study away credit will be granted toward the major or concentration.

**Can your department or program typically pre-approve courses for major/concentration credit?**

Yes, in many cases, though students should be sure to contact the department.

**What criteria will typically be used/required to determine whether a student may receive major/concentration credit for a course taken while on study away?**

Course title and description, and complete syllabus including readings/assignments.

**Does your department/program place restrictions on the number of major/concentration credits that a student might earn through study away?**

No.

**Does your department/program place restrictions on the types of courses that can be awarded credit towards your major?**

Yes. They have to be approved MATH/STAT courses.

**Are there specific major requirements that cannot be fulfilled while on study away?**

Yes. Colloquium requirement, Senior Seminar requirement.

**Are there specific major requirements in your department/program that students should be particularly aware of when weighing study away options? (Some examples might include a required course that is always taught in one semester, laboratory requirements.)**

Yes. The highly cumulative structure of the major.

**Give examples in which students thought or assumed that courses taken away would count toward the major or concentration and then learned they wouldn’t:**

None to date.

**THE DEGREE WITH HONORS IN MATHEMATICS**

The degree with honors in Mathematics is awarded to the student who has demonstrated outstanding intellectual achievement in a program of study which extends beyond the requirements of the major. The principal considerations for recommending a student for the degree with honors will be: Mastery of core material and skills, breadth and, particularly, depth of knowledge beyond the core material, ability to pursue independent study of mathematics or statistics, originality in methods of investigation, and, where appropriate, creativity in research.

An honors program normally consists of two semesters (MATH/STAT 493 and 494) and a winter study (WSP 031) of independent research, culminating in a thesis and a presentation. Under certain circumstances, the honors work can consist of coordinated study involving a one semester (MATH/STAT 493 or 494) and a winter study (WSP 030) of independent research, culminating in a “minithesis” and a presentation. Note that during the Academic Year 2020-2021 the winter study requirement for thesis and “minithesis” is waved. At least one semester should be in addition to the major requirements, and thesis courses do not count as 400-level senior seminars.

An honors program in actuarial studies requires significant achievement on four appropriate examinations of the Society of Actuaries.

Highest honors will be reserved for the rare student who has displayed exceptional ability, achievement or originality. Such a student usually will have written a thesis, or pursued actuarial honors and written a mini-thesis. An outstanding student who writes a mini-thesis, or pursues actuarial honors and writes a paper, might also be considered. In all cases, the award of honors and highest honors is the decision of the Department.

**APPLIED MATHEMATICS TRACK**

Students interested in applied mathematics, engineering, or other sciences should consider:

- Mathematics 140 Calculus II
- Mathematics 150 or 151 Multivariable Calculus
- Statistics 201 Statistics and Data Analysis
Mathematics 250 Linear Algebra
Mathematics 351 Applied Real Analysis
Mathematics 355 Abstract Algebra
Some programming or numerical analysis (e.g. MATH 361, 318T, or anything if you’ve had CSCI 134)
MATH 309 or Post-core Differential Equations/Numerical Methods
Senior seminar (e.g. Math Ecology MATH 410T or Mathematical Modeling MATH 433)

Other recommended courses: complex analysis, discrete mathematics (e.g. combinatorics or graph theory), operations research, optimization, probability, statistics, appropriate courses in Biology, Chemistry, Computer Science, Economics, Neuroscience, Physics, etc.

Williams has exchange and joint programs with good engineering schools. Interested students should consult the section on engineering near the beginning of the Bulletin and the Williams pre-engineering advisor for further information.

GRADUATE SCHOOL TRACK
Students interested in continuing their study of mathematics in graduate school should consider:

Mathematics 140 Calculus II
Mathematics 150 or 151 Multivariable Calculus
Mathematics 250 Linear Algebra
Mathematics 350 Real Analysis
Mathematics 355 Abstract Algebra
Complex Analysis
Topology
Some second semester analysis
Some second semester algebra
Some post-core geometry
Thesis

[With prior permission, in unusual circumstances, senior seminar can be waived in favor of harder post-core electives.]

Students headed for graduate school generally take more than this relatively small number of courses required for a liberal arts major. Reading knowledge of a foreign language (French, German, or Russian) can be helpful.

Students interested in studying statistics in graduate school should take STAT 201, 346, 360, a 400 level statistics course and MATH 350/351 and 341 in addition to their other math requirements.

OTHER CAREER PATHS

Other Graduate and Professional Schools: An increasing number of graduate and professional schools require mathematics and statistics as a prerequisite to admission or to attaining their degree. Students interested in graduate or professional training in business, medicine, economics, or psychology are advised to find out the requirements in those fields early in their college careers.

Statistics and Actuarial Science: Students interested in statistics or actuarial science should consider Mathematics 341, Statistics courses, and Economics 255. Additionally, students should consider taking some number of the actuarial exams given by the Society of Actuaries, which can constitute part of an honors program in actuarial studies (see section on honors above).

Teaching: Students interested in teaching mathematics at the elementary or secondary school level should consider courses on teaching, number theory, geometry, statistics, and practice as a tutor or teaching assistant. Winter study courses that provide a teaching practicum are also highly recommended. Consult the Program in Teaching (Professor Susan Engel) and the Office of Career Counseling.

Business and Finance: Students interested in careers in business or finance should consider Mathematics 333 and Statistics courses. Since these courses address different needs, students should consult with the instructors to determine which seem to be most appropriate for individuals.

There are three types of 300-level courses. There are the core courses: Real Analysis, MATH 350/351, and Abstract Algebra Math 355. There are the "precore" courses, which do not have the core courses as prerequisites and have numbers 300-349. Finally, there are those courses that have an Abstract Algebra or Real Analysis prerequisite, which are numbered 360-399.
MATH 102  (F)  Foundations in Quantitative Skills
This course will strengthen a student's foundation in quantitative reasoning in preparation for the science curriculum and QFR requirements. The material will be at the college algebra/precalculus level, and covered in a tutorial format with students working in small groups with the professor. Access to this course is limited to placement by a quantitative skills counselor.

Requirements/Evaluation: homework, presentations during the tutorial meetings, and projects
Prerequisites: access to the course is limited to placement by a quantitative skills counselor
Enrollment Limit: 10
Enrollment Preferences: students who need most help with the quantitative reasoning
Expected Class Size: 10
Grading: no pass/fail option, no fifth course option
Distributions: (D3)

Fall 2020
TUT Section: HT1    TBA     Mihai  Stoiciu

MATH 110  (F)  Logic and Likelihood  (QFR)
How best can we reason in the face of uncertainty? We will begin with an examination of rationality and the reasoning process including a survey of formal logic. Starting with uncertainty from a psychological and philosophical viewpoint, we will move to a careful theory of likelihood and how to reason with probabilistic models. The course will conclude with a consideration of observation and information, how to test hypotheses, and how we update our beliefs to incorporate new evidence.

Requirements/Evaluation: homework, essays, presentations, exams, and participation
Prerequisites: none
Enrollment Limit: 25
Enrollment Preferences: first-year students
Expected Class Size: 25
Grading: no pass/fail option, no fifth course option
Distributions: (D3)  (QFR)
Quantative/Formal Reasoning Notes: This course will be covering formal logic and probability theory at sufficient depth to place this course on level with other QFR designated courses.
Not offered current academic year

MATH 113  (S)  The Beauty of Numbers  (QFR)
Have you ever wondered what keeps your credit card information safe everytime you buy something online? Number theory! Number Theory is one of the oldest branches of mathematics. In this course, we will discover the beauty and usefulness of numbers, from ancient Greece to modern cryptography. We will look for patterns, make conjectures, and learn how to prove these conjectures. Starting with nothing more than basic high school algebra, we will develop the logic and critical thinking skills required to realize and prove mathematical results. Topics to be covered include the meaning and content of proof, prime numbers, divisibility, rationality, modular arithmetic, Fermat's Last Theorem, the Golden ratio, Fibonacci numbers, coding theory, and unique factorization.

Requirements/Evaluation: projects, homework assignments, and exams
Prerequisites: MATH 102 (or demonstrated proficiency on a diagnostic test) or permission of instructor
Enrollment Limit: 25
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)  (QFR)

Not offered current academic year

**MATH 119  (F)  The Mathematics of Pandemics: From the Spread of Infections to Cost-Benefit Analyses of Responses  (QFR)**

The goal of the class is to help students learn to ask the right questions, and to gather and analyze the data needed to answer them, to understand the covid pandemic and the worldwide responses. Through local experts and numerous guest speakers playing key roles in these problems, we will discuss numerous aspects, from mathematical models for virus propagation to analyzing the economic, educational, social and emotional consequences of lockdowns and social distancing; from moral and legal dilemmas created by the pandemic and responses to the international political scene and relations between countries. Offered as Math 119 or Math 312 (those taking as Math 312 will have some of the readings replaced with more technical modeling papers and subsequent homework). Pre-requisites: None for Math 119; for Math 312 linear algebra is recommended.

**Requirements/Evaluation:** Homework, writing, class participation.

**Prerequisites:** none

**Enrollment Limit:** 30

**Enrollment Preferences:** all students will have an equal chance; if possible none will be turned away.

**Expected Class Size:** 30

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3)  (QFR)

**Quantative/Formal Reasoning Notes:** We will discuss mathematical models and use statistics to analyze data.

Fall 2020

LEC Section: H1  MWF 10:40 am - 11:30 am  Steven J. Miller

**MATH 130  (F)(S)  Calculus I  (QFR)**

Calculus permits the computation of velocities and other instantaneous rates of change by a limiting process called differentiation. The same process also solves "max-min" problems: how to maximize profit or minimize pollution. A second limiting process, called integration, permits the computation of areas and accumulations of income or medicines. The Fundamental Theorem of Calculus provides a useful and surprising link between the two processes. Subtopics include trigonometry, exponential growth, and logarithms.

**Requirements/Evaluation:** Weekly homework and quizzes, 2 exams during the semester, and one final

**Prerequisites:** MATH 102 (or demonstrated proficiency on a diagnostic test); this is an introductory course for students who have not seen calculus before

**Enrollment Limit:** 30

**Enrollment Preferences:** first-year students

**Expected Class Size:** 20

**Grading:** yes pass/fail option, yes fifth course option

**Unit Notes:** students who have previously taken a calculus course may not enroll in MATH 130 without the permission of instructor

**Distributions:** (D3)  (QFR)

**Quantative/Formal Reasoning Notes:** This a calculus course.

Fall 2020

LEC Section: H1  TF 1:30 pm - 2:45 pm  Lori A. Pedersen

LEC Section: H2  TF 3:15 pm - 4:30 pm  Lori A. Pedersen

Spring 2021

LEC Section: H1  MWF 9:20 am - 10:10 am  Lori A. Pedersen

**MATH 140  (F)(S)  Calculus II  (QFR)**
Mastery of calculus requires understanding how integration computes areas and business profit and acquiring a stock of techniques. Further methods solve equations involving derivatives ("differential equations") for population growth or pollution levels. Exponential and logarithmic functions and trigonometric and inverse functions play an important role. This course is the right starting point for students who have seen derivatives, but not necessarily integrals, before.

Requirements/Evaluation: homework, quizzes, and/or exams

Prerequisites: MATH 130 or equivalent; students who have received the equivalent of advanced placement of AB 4, BC 3 or higher may not enroll in MATH 140 without the permission of instructor

Enrollment Limit: 30

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Unit Notes: students who have higher advanced placement must enroll in MATH 150 or above

Distributions: (D3) (QFR)

Fall 2020
LEC Section: R1  TF 1:30 pm - 2:45 pm  Josh Carlson
LEC Section: R2  TF 3:15 pm - 4:30 pm  Josh Carlson

Spring 2021
LEC Section: R1  TR 9:45 am - 11:00 am  Josh Carlson

MATH 150 (F)(S) Multivariable Calculus (QFR)

Applications of calculus in mathematics, science, economics, psychology, the social sciences, involve several variables. This course extends calculus to several variables: vectors, partial derivatives, multiple integrals. There is also a unit on infinite series, sometimes with applications to differential equations. Note: This course will be taught via flipped-course method, an instructional strategy reversing the traditional learning environment by delivering instructional content outside of the classroom. This includes prerecorded lectures along with questions that students must watch and answer prior to attending class. Class time include synchronous meetings clarifying concepts and working in small groups through challenging problems with the support of the professor and peers. Building positive collaborative working relationships and public speaking skills will be added benefits of this course.

Requirements/Evaluation: Video readiness assessments, problem sets, exams, and participation.

Prerequisites: MATH 140 or equivalent, such as satisfactory performance on an Advanced Placement Examination

Enrollment Limit: 30

Enrollment Preferences: Professor's discretion

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Unit Notes: students with the equivalent of advanced placement of AB 4 or above should enroll in MATH 150, students with a BC 3 or higher should enroll in Math 151 when it is being offered, and Math 150 otherwise.

Distributions: (D3) (QFR)

Quantative/Formal Reasoning Notes: mathematics

Fall 2020
SEM Section: R1  TR 8:00 am - 9:15 am  Pamela E. Harris
SEM Section: R2  TR 9:45 am - 11:00 am  Pamela E. Harris
SEM Section: R3  TR 11:30 am - 12:45 pm  Pamela E. Harris

Spring 2021
SEM Section: H1  MWF 9:20 am - 10:10 am  Steven J. Miller
SEM Section: H2  MWF 10:40 am - 11:30 am  Steven J. Miller
Applications of calculus in mathematics, science, economics, psychology, the social sciences, involve several variables. This course extends calculus to several variables: vectors, partial derivatives and multiple integrals. The goal of the course is Stokes Theorem, a deep and profound generalization of the Fundamental Theorem of Calculus. The difference between this course and MATH 150 is that MATH 150 covers infinite series instead of Stokes Theorem. Students with the equivalent of BC 3 or higher should enroll in MATH 151, as well as students who have taken the equivalent of an integral calculus and who have already been exposed to infinite series. For further clarification as to whether MATH 150 or MATH 151 is appropriate, please consult a member of the math/stat department.

Class Format: Hybrid; short lectures will be asynchronous, with longer synchronous in-person problem sessions (these will be available remotely, and uploaded later for asynchronous viewing)

Requirements/Evaluation: homework, quizzes, and exams

Prerequisites: AP BC 3 or higher or integral calculus with infinite series

Enrollment Limit: 30

Enrollment Preferences: First-years, sophomores, and juniors

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Unit Notes: MATH 151 satisfies any MATH 150 prerequisite; credit will not be given for both MATH 150 and MATH 151

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This course builds quantitative skills

Fall 2020
LEC Section: H1 MWF 9:20 am - 10:10 am Ralph E. Morrison
LEC Section: H2 MWF 10:40 am - 11:30 am Ralph E. Morrison
LEC Section: H3 MWF 12:00 pm - 12:50 pm Ralph E. Morrison

The fundamental goal of this course is for students to acquire the ability to create and clearly express mathematical arguments through an exploration of topics from discrete mathematics. Students will learn various mathematical proof techniques while discovering such areas as logic, number theory, infinity, graph theory, and probability. A large component of the class is focused on problem solving and proof writing skills. The format of the course during the Spring 2021 semester will be a combination of lecture and discovery based learning. Students will attend remote synchronous lectures once a week. They will also have weekly small group meetings (30 minutes) with a TA and other classmates, and work through some course material independently.

Class Format: The format of the course during the Spring 2021 semester will be a combination of lecture and discovery based learning with weekly small group meetings.

Requirements/Evaluation: Evaluation will be based primarily on homework, exams, and group meeting assignments.

Prerequisites: Calculus at the level of an AP course or Williams College Math 130 or 140. Students who have taken a 300-level or 400-level math course should obtain permission of the instructor before enrolling.

Enrollment Limit: 30

Enrollment Preferences: As determined by instructor.

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This course involve developing the formal mathematical language of logic and set theory. It also involves using quantitative tools to solve problems relating to combinatorics, probability, and other fields of discrete mathematics.

Fall 2020
LEC Section: R1 TR 9:45 am - 11:00 am Chad M. Topaz
MATH 210 (S) Mathematical Methods for Scientists (QFR)

Cross-listings: PHYS 210 MATH 210

Secondary Cross-listing

This course covers a variety of mathematical methods used in the sciences, focusing particularly on the solution of ordinary and partial differential equations. In addition to calling attention to certain special equations that arise frequently in the study of waves and diffusion, we develop general techniques such as looking for series solutions and, in the case of nonlinear equations, using phase portraits and linearizing around fixed points. We study some simple numerical techniques for solving differential equations. A series of optional sessions in Mathematica will be offered for students who are not already familiar with this computational tool.

Class Format: three hours per week; hybrid course format

Requirements/Evaluation: several exams and on weekly problem sets, all of which have a substantial quantitative component

Prerequisites: MATH 150 or 151 and familiarity with Newtonian mechanics at the level of PHYS 131

Enrollment Limit: 50

Enrollment Preferences: sophomores and juniors

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:

PHYS 210 (D3) MATH 210 (D3)

Quantitative/Formal Reasoning Notes: This course will have weekly problem sets using advanced calculus methods and some computer programming at the end of the course.

Spring 2021

LEC Section: H1 TR 11:30 am - 12:45 pm Frederick W. Strauch

MATH 250 (F)(S) Linear Algebra (QFR)

Many social, political, economic, biological, and physical phenomena can be described, at least approximately, by linear relations. In the study of systems of linear equations one may ask: When does a solution exist? When is it unique? How does one find it? How can one interpret it geometrically? This course develops the theoretical structure underlying answers to these and other questions and includes the study of matrices, vector spaces, linear independence and bases, linear transformations, determinants and inner products. Course work is balanced between theoretical and computational, with attention to improving mathematical style and sophistication.

Class Format: Unless circumstances change, students will have the option of taking the course in person or remotely.

Requirements/Evaluation: homework, exams, and possibly short remote meetings outside of class.

Prerequisites: MATH 150/151 or MATH 200

Enrollment Limit: 30

Enrollment Preferences: Students who have officially declared a major that requires Math 250.

Expected Class Size: 30

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: In this course, students will engage in both quantitative and formal reasoning.

Attributes: COGS Related Courses
MATH 306 (S) Fractals and Chaos  (QFR)

Early in the course we introduce the notion of dynamical systems. Then we will develop the mathematics behind iterated function systems and study the notions of fractals and chaos. There will be a lot of computer experimentation with various programs and resources which the students are expected to use to learn and discover properties of fractals. The final topics will include dimension complex dynamics and the Mandelbrot set.

Requirements/Evaluation:  homework, projects and exams
Prerequisites:  MATH 250
Enrollment Limit:  30
Expected Class Size:  18
Grading:  yes pass/fail option,  yes fifth course option
Distributions:  (D3)  (QFR)

Not offered current academic year

MATH 307 (S) Computational Linear Algebra  (QFR)

Linear algebra is of central importance in the quantitative sciences, including application areas such as image and signal processing, data mining, computational finance, structural biology, and much more. When the problems must be solved computationally, approximation, round-off errors, convergence, and efficiency matter, and traditional linear algebra techniques may fail to succeed. We will adopt linear algebra techniques on a large scale, implement them computationally, and apply them to core problems in scientific computing. Topics may include: systems of linear and nonlinear equations; approximation and statistical function estimation; optimization; interpolation; data scraping; singular value decomposition; and more. This course could also be considered a course in numerical analysis or computational science.

Class Format: To afford students flexibility during the COVID pandemic, this course is taught online. Students will read and/or watch lecture material asynchronously and will participate in a once-per-week synchronous small-group tutorial meeting with the instructor via video chat. This course will be a good fit for students with a strong interest in applied mathematics and a willingness to devote significant effort to learning/doing computer programming.

Requirements/Evaluation: Students will complete checkpoint quizzes, regularly assigned homework problems and projects, and reflective writing assignments. To move towards a non-hierarchical, transparent, and egalitarian grading system, the instructor follows the policy of "ungrading." Over the course of the semester, students will develop a rubric to assess their own learning and will evaluate themselves according to this rubric.
Prerequisites:  MATH 250; some prior exposure to computer programming experience is strongly recommended but not required.
Enrollment Limit:  20
Enrollment Preferences:  Professor's discretion
Expected Class Size:  20
Grading:  yes pass/fail option,  yes fifth course option
Distributions:  (D3)  (QFR)

Quantitative/Formal Reasoning Notes: This course involves developing the formal mathematical language of linear algebra. It also involves using quantitative tools to solve problems relating to a wide range of applications in the physical and social sciences.

Spring 2021
LEC Section: R1   TR 9:45 am - 11:00 am   Chad M. Topaz

MATH 309 (F) Differential Equations  (QFR)
Ordinary differential equations (ODE) frequently arise as models of phenomena in the natural and social sciences. This course presents core ideas of ODE from an applied standpoint. Topics covered early in the course may include numerical solutions, separation of variables, integrating factors, constant coefficient linear equations, and power series solutions. Later, we will focus on nonlinear ODEs, for which it is usually impossible to find analytical solutions. Tools from dynamical systems will be introduced to allow us to obtain some information about the behavior of the ODE without explicitly knowing the solution.

Class Format: Unless circumstances change, students will have the option of taking the course in person or remotely
Requirements/Evaluation: quizzes/exams, problem sets, participation, and possible activities
Prerequisites: MATH 150/151 and MATH 250
Enrollment Limit: 20
Enrollment Preferences: discretion of the instructor
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: 300-level mathematics course

Fall 2020
LEC Section: R1 TR 9:45 am - 11:00 am Julie C. Blackwood

MATH 310 (F) Mathematical Biology (QFR)
Cross-listings: BIOL 210 MATH 310
Primary Cross-listing
This course will provide an introduction to the many ways in which mathematics can be used to understand, analyze, and predict biological dynamics. We will learn how to construct mathematical models that capture essential properties of biological processes while maintaining analytic tractability. Analytic techniques, such as stability and bifurcation analysis, will be introduced in the context of both continuous and discrete time models. Additionally, students will couple these analytic tools with numerical simulation to gain a more global picture of the biological dynamics. Possible biological applications include, but are not limited to, single and multi-species population dynamics, neural and biological oscillators, tumor cell growth, and infectious disease dynamics.
Class Format: Unless circumstances change, students will have the option of taking the course in person or remotely
Requirements/Evaluation: problem sets, quizzes/exams, participation, final project and paper
Prerequisites: MATH 250 and MATH 309, or permission of instructor
Enrollment Limit: 20
Enrollment Preferences: if over-enrolled, will have students submit reasons for taking class; preference to those with interest in both subjects
Expected Class Size: 20
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)
This course is cross-listed and the prefixes carry the following divisional credit:
BIOL 210 (D3) MATH 310 (D3)
Quantitative/Formal Reasoning Notes: The course will introduce methods for developing and analyzing mathematical models.
Attributes: PHLH Methods in Public Health

Fall 2020
LEC Section: R1 TR 11:30 am - 12:45 pm Julie C. Blackwood

MATH 312 (F) The Mathematics of Pandemics: From the Spread of Infections to Cost-Benefit Analyses of Responses (QFR)
The goal of the class is to help students learn to ask the right questions, and to gather and analyze the data needed to answer them, to understand the covid pandemic and the worldwide responses. Through local experts and numerous guest speakers playing key roles in these problems, we will
discuss numerous aspects, from mathematical models for virus propagation to analyzing the economic, educational, social and emotional consequences of lockdowns and social distancing; from moral and legal dilemmas created by the pandemic and responses to the international political scene and relations between countries. Offered as Math 119 or Math 312 (those taking as Math 312 will have some of the readings replaced with more technical modeling papers and subsequent homework). Pre-requisites: None for Math 119; for Math 312 linear algebra is recommended.

Requirements/Evaluation: Class participation, writing, homework problems.
Prerequisites: Linear algebra recommended.
Enrollment Limit: none
Enrollment Preferences: all students will have an equal chance; if possible none will be turned away.
Expected Class Size: 30
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantative/Formal Reasoning Notes: We will discuss mathematical models and use statistics to analyze data.

Fall 2020
LEC Section: H1  MWF 10:40 am - 11:30 am  Steven J. Miller

MATH 313  (S) Introduction to Number Theory  (QFR)
The study of numbers dates back thousands of years, and is fundamental in mathematics. In this course, we will investigate both classical and modern questions about numbers. In particular, we will explore the integers, and examine issues involving primes, divisibility, and congruences. We will also look at the ideas of numbers and primes in more general settings, and consider fascinating questions that are simple to understand, but can be quite difficult to answer. This course will be held virtually using an active learning method, an instructional strategy reversing the traditional learning environment by supplying instructional content outside of class time. This will include reading the textbook and completing problem sets prior to attending class. Class time will be spent clarifying concepts and working in small groups through challenging problems with the support of the professor, teaching assistants, and your peers. Building positive collaborative working relationships and public speaking skills will be added benefits of this class.

Class Format: This course will employ an active learning method rather than the traditional lecture. Please see the course description for details.
Requirements/Evaluation: The course will be graded on a mastery-based system. The final course grade will be a combination of quarterly participation in self-reflections, daily reading assignments, and weekly problem sets.
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 20
Enrollment Preferences: All are welcome regardless of major or year. In case of over-enrollment, preference will be given to those needing the course for graduation.
Expected Class Size: 15
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantative/Formal Reasoning Notes: This course requires working with various number systems, performing explicit computations, and proving mathematical results using logical reasoning practices.

Spring 2021
LEC Section: R1  TF 1:30 pm - 2:45 pm  Eva Goedhart

MATH 314  (S) Cryptography  (QFR)
An introduction to the techniques and practices used to keep secrets over non-secure lines of communication, including classical cryptosystems, the data encryption standard, the RSA algorithm, discrete logarithms, hash functions, and digital signatures. In addition to the specific material, there will also be an emphasis on strengthening mathematical problem solving skills, technical reading, and mathematical communication.

Requirements/Evaluation: exams, homework, and quizzes
Prerequisites: MATH 250
Enrollment Limit: 30
Enrollment Preferences: graduating seniors and Math majors
Expected Class Size: 30
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: The course will contain mathematical proofs.
Not offered current academic year

MATH 315  (S)  Methods for Solving Diophantine Equations  (QFR)
A Diophantine equation is an equation with integer (or rational) coefficients that is to be solved in integers (or rational numbers). A focus of study for hundreds of years, Diophantine analysis remains a vibrant area of research. It has yielded a multitude of beautiful results and has wide ranging applications in other areas of mathematics, in cryptography, and in the natural sciences. In this project-based tutorial, we will focus on studying and implementing various methods for solving previously unsolved infinite families of Diophantine equations. Depending on their interests, students may choose one or several methods to apply to open problems in the field. Please note that this tutorial will be held virtually.
Requirements/Evaluation: The grade for this course will be a combination of weekly problem sets, weekly oral presentations (approx. 15 min. each), quarterly self-reflections, and a final written project manuscript that will be continually edited throughout the semester (minimum of 5 pages).
Prerequisites: MATH 250 or permission of the instructor
Enrollment Limit: 10
Enrollment Preferences: Sophomores, Juniors, and Seniors based on a short questionnaire of interests. In the event of over-enrollment, preference will be given to those that need the course to graduate.
Expected Class Size: 10
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This course requires working with various number systems, performing explicit computations, and proving mathematical results using logical reasoning practices.

Spring 2021
TUT Section: RT1    TBA     Eva  Goedhart

MATH 317  (F)  Introduction to Operations Research  (QFR)
In the first N math classes of your career, you can be misled as to what the world is truly like. How? You're given exact problems and told to find exact solutions. The real world is sadly far more complicated. Frequently we cannot exactly solve problems; moreover, the problems we try to solve are themselves merely approximations to the world! We are forced to develop techniques to approximate not just solutions, but even the statement of the problem. Additionally, we often need the solutions quickly. Operations Research, which was born as a discipline during the tumultuous events of World War II, deals with efficiently finding optimal solutions. In this course we build analytic and programming techniques to efficiently tackle many problems. We will review many algorithms from earlier in your mathematical or CS career, with special attention now given to analyzing their run-time and seeing how they can be improved. The culmination of the course is a development of linear programming and an exploration of what it can do and what are its limitations. For those wishing to take this as a Stats course, the final project must have a substantial stats component approved by the instructor.
Prerequisites: Linear Algebra (MATH 250) and one other 200-level or higher CSCI, MATH or STATS course.
Requirements/Evaluation: homework, exams, projects
Prerequisites: MATH 150, MATH 250 and one other 200-level or higher CSCI, MATH or STATS course
Enrollment Limit: 40
Enrollment Preferences: Computer Science, Mathematics and Statistics majors
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Unit Notes: http://web.williams.edu/Mathematics/sjmiller/public_html/317/
Distributions: (D3) (QFR)
MATH 319  (S)  Integrative Bioinformatics, Genomics, and Proteomics Lab  (QFR)

Cross-listings:  MATH 319  CHEM 319  BIOL 319  PHYS 319  CSCI 319

Secondary Cross-listing

What can computational biology teach us about cancer? In this lab-intensive experience for the Genomics, Proteomics, and Bioinformatics program, computational analysis and wet-lab investigations will inform each other, as students majoring in biology, chemistry, computer science, mathematics/statistics, and physics contribute their own expertise to explore how ever-growing gene and protein data-sets can provide key insights into human disease. In this course, we will take advantage of one well-studied system, the highly conserved Ras-related family of proteins, which play a central role in numerous fundamental processes within the cell. The course will integrate bioinformatics and molecular biology, using database searching, alignments and pattern matching, and phylogenetics to reconstruct the evolution of gene families by focusing on the gene duplication events and gene rearrangements that have occurred over the course of eukaryotic speciation. By utilizing high through-put approaches to investigate genes involved in the inflammatory and MAPK signal transduction pathways in human colon cancer cell lines, students will uncover regulatory mechanisms that are aberrantly altered by siRNA knockdown of putative regulatory components. This functional genomic strategy will be coupled with independent projects using phosphorylation-state specific antisera to test our hypotheses. Proteomic analysis will introduce the students to de novo structural prediction and threading algorithms, as well as data-mining approaches and Bayesian modeling of protein network dynamics in single cells. Flow cytometry and mass spectrometry may also be used to study networks of interacting proteins in colon tumor cells.

Class Format: two afternoons of lab, with one hour of lecture, per week. In most weeks, we will meet one day for lecture discussions.

Requirements/Evaluation:  lab participation, several short homework assignments, one lab report, a programming project, and a grant proposal

Prerequisites:  BIOL 202; students who have not taken BIOL 202 but have taken BIOL 101 and a CSCI course, or CSCI/PHYS 315, may enroll with permission of instructor. No prior computer programming experience is required.

Enrollment Limit:  12

Enrollment Preferences:  seniors, then juniors, then sophomores

Expected Class Size:  12

Grading:  yes pass/fail option,  yes fifth course option

Unit Notes:  does not satisfy the distribution requirement for the Biology major

Distributions:  (D3)  (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:

MATH 319 (D3) CHEM 319 (D3) BIOL 319 (D3) PHYS 319 (D3) CSCI 319 (D3)

Quantative/Formal Reasoning Notes:  Through lab work, homework sets and a major project, students will learn or further develop their skills in programming in Python, and about the basis of Bayesian approaches to phylogenetic tree estimation.

Attributes:  BIGP Courses  BIMO Interdepartmental Electives

Spring 2021

SEM Section: 01  TR 9:45 am - 11:00 am  Lois M. Banta
LAB Section: H3  MW 1:00 pm - 3:00 pm  Lois M. Banta
LAB Section: H4  TR 1:00 pm - 3:00 pm  Lois M. Banta
LAB Section: H5
SEM Section: R2  MW 6:45 pm - 8:00 pm  Lois M. Banta

MATH 321  (S)  Knot Theory  (QFR)

Take a piece of string, tie a knot in it, and glue the ends together. The result is a knotted circle, known as a knot. For the last 100 years, mathematicians have studied knots, asking such questions as, "Given a nasty tangled knot, how do you tell if it can be untangled without cutting it open?" Some of the most interesting advances in knot theory have occurred in the last ten years. This course is an introduction to the theory of knots. Among other topics, we will cover methods of knot tabulation, surfaces applied to knots, polynomials associated to knots, and relationships between knot theory and chemistry and physics. In addition to learning the theory, we will look at open problems in the field.
Requirements/Evaluation: problem sets, midterms, a paper and a final exam
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 30
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)

Not offered current academic year

MATH 325  (F)  Set Theory  (QFR)
Set theory is the traditional foundational language for all of mathematics. We will be discussing the Zermelo-Fraenkel axioms, including the Axiom of Choice and the Continuum Hypothesis, basic independence results and, if time permits, incompleteness theorems. At one time, these issues tore at the foundations of mathematics. They are still vital for understanding the nature of mathematical truth.
Requirements/Evaluation: exams and homework
Prerequisites: MATH 250
Enrollment Limit: 30
Expected Class Size: 15
Grading: yes pass/fail option, yes fifth course option
Materials/Lab Fee: textbook cost
Distributions: (D3) (QFR)

Not offered current academic year

MATH 328  (S)  Combinatorics  (QFR)
Combinatorics is a branch of mathematics that focuses on enumerating, examining, and investigating the existence of discrete mathematical structures with certain properties. This course provides an introduction to the fundamental structures and techniques in combinatorics including enumerative methods, generating functions, partition theory, the principle of inclusion and exclusion, and partially ordered sets.
Class Format: interactive activities and discussion
Requirements/Evaluation: quizzes/exams, homework, activities
Prerequisites: MATH 200 and MATH 250
Enrollment Limit: 30
Enrollment Preferences: discretion of the instructor
Expected Class Size: 25
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)

Not offered current academic year

MATH 329  (S)  Discrete Geometry  (QFR)
Discrete geometry is one of the oldest and most consistently vibrant areas of mathematics, stretching from the Platonic Solids of the ancient Greeks to the modern day applications of convex optimization and linear programming. In this tutorial we will learn about polygons and their higher-dimensional cousins, polyhedra and polytopes, and the various ways to describe, compute, and classify such objects. We will learn how these objects and ideas can be applied to other areas, from computation and optimization to studying areas of math like algebraic geometry. Throughout this course we will be engaging with mathematical work and literature from as old as 500 BCE and as recent as "posted to the internet yesterday."
Requirements/Evaluation: participation, problem sets, oral presentations, an oral exam, and a final project
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 10
Enrollment Preferences: first-years and sophomores
MATH 331  (F) The little Questions  (QFR)
Using math competitions such as the Putnam Exam as a springboard, in this class we follow the dictum of the Ross Program and “think deeply of simple things”. The two main goals of this course are to prepare students for competitive math competitions, and to get a sense of the mathematical landscape encompassing elementary number theory, combinatorics, graph theory, and group theory (among others). While elementary frequently is not synonymous with easy, we will see many beautiful proofs and “a-ha” moments in the course of our investigations. Students will be encouraged to explore these topics at levels compatible with their backgrounds.

Requirements/Evaluation: homework, classwork, and exams
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 30
Enrollment Preferences: members or alternates of the Putnam team, Mathematics, Physics or Computer Science majors
Expected Class Size: 25
Grading: yes pass/fail option, yes fifth course option
Unit Notes: http://web.williams.edu/Mathematics/sjmiller/public_html/331/
Distributions: (D3) (QFR)
Not offered current academic year

MATH 334  (S) Graph Theory  (QFR)
A graph is a collection of vertices, joined together by edges. In this course, we will study the sorts of structures that can be encoded in graphs, along with the properties of those graphs. We’ll learn about such classes of graphs as multi-partite, planar, and perfect graphs, and will see applications to such optimization problems as minimum colorings of graphs, maximum matchings in graphs, and network flows.

Requirements/Evaluation: problem sets and exams
Prerequisites: MATH 200 or MATH 250
Enrollment Limit: 20
Enrollment Preferences: Math majors
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantative/Formal Reasoning Notes: This course involves the writing of mathematical proofs.

Spring 2021
LEC Section: R1    TR 11:30 am - 12:45 pm    Josh Carlson

MATH 337  (F) Electricity and Magnetism for Mathematicians  (QFR)
Maxwell's equations are four simple formulas, linking electricity and magnetism, that are among the most profound equations ever discovered. These equations led to the prediction of radio waves, to the realization that a description of light is also contained in these equations and to the discovery of the special theory of relativity. In fact, almost all current descriptions of the fundamental laws of the universe are deep generalizations of Maxwell's equations. Perhaps even more surprising is that these equations and their generalizations have led to some of the most important mathematical discoveries (where there is no obvious physics) of the last 25 years. For example, much of the math world was shocked at how these physics generalizations became one of the main tools in geometry from the 1980s until today. It seems that the mathematics behind Maxwell is endless. This will be an introduction to Maxwell's equations, from the perspective of a mathematician.

Requirements/Evaluation: performance on homework and exams
Prerequisites: MATH 250; no physics background required

Enrollment Limit: none

Enrollment Preferences: none

Expected Class Size: 15

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Not offered current academic year

MATH 338 (F) Intermediate Logic (QFR)

Cross-listings: MATH 338 PHIL 338

Secondary Cross-listing

In this course, we will begin with an in-depth study of the theory of first-order logic. We will first get clear on the formal semantics of first-order logic and various ways of thinking about formal proof: natural deduction systems, semantic tableaux, axiomatic systems and sequent calculi. Our main goal will be to prove things about this logical system rather than to use this system to think about ordinary language arguments. In this way the goal of the course is significantly different from that of Logic and Language (PHIL 203). Students who have take PHIL 203 will have a good background for this class, but students who are generally comfortable with formal systems need not have taken PHIL 203. We will prove soundness and completeness, compactness, the Lowenheim-Skolem theorems, undecidability and other important results about first-order logic. As we go through these results, we will think about the philosophical implications of first-order logic. From there, we will look at extensions of and/or alternatives to first-order logic. Possible additional topics would include: modal logic, the theory of counterfactuals, alternative representations of conditionals, the use of logic in the foundations of arithmetic and Godel's Incompleteness theorems. Student interest will be taken into consideration in deciding what additional topics to cover.

Requirements/Evaluation: problem sets and exams

Prerequisites: some class in which student has studied formal reasoning

Enrollment Limit: 20

Enrollment Preferences: Philosophy majors; juniors and seniors

Expected Class Size: 15

Grading: yes pass/fail option, yes fifth course option

Distributions: (D2) (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:

MATH 338 (D3) PHIL 338 (D2)

Quantitative/Formal Reasoning Notes: This is a class in Formal Logic. PHIL 203 satisfies the QFR requirement. If anything, this class will be significantly more formal.

Attributes: Linguistics

Not offered current academic year

MATH 340 Applications of Mathematics to the Real World (QFR)

Often for real world applications one does not need to find the optimal solution, which can be extremely difficult, but instead just find something close, or at least better than what is currently being done. We will develop material and techniques from mathematics, statistics and allied fields with an eye to applications. In addition to standard homework assignments and exams there will be a group project where students will work with a local business, write a report and present the results. Pre-requisites are multivariable calculus and linear algebra, or permission of the instructor. Knowledge of some statistics or programming is beneficial but not required.

Class Format: In addition to standard lectures and assignments, we will be partnering with local businesses to apply mathematics to solve real world problems.

Requirements/Evaluation: Lectures and class participation, homework, exams and encouragement to do a project. We already have several local businesses with projects for students. Working on a project will entail meeting with officials from the company, clearly defining what the problem is, and writing a solution. This will include a presentation, a write-up, and potentially implementable code. Based on previous similar courses, these papers typically run from 10 to 40 pages.
Prerequisites: Mathematics 150 or 151, and Linear Algebra, or permission of the instructor.

Enrollment Limit: 40

Enrollment Preferences: Students who have taken at least one statistics or computer science class

Expected Class Size: 25

Grading:

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is a 300 level mathematics class

Not offered current academic year

MATH 341 (F)(S) Probability (QFR)

Cross-listings: STAT 341  MATH 341

Primary Cross-listing

The historical roots of probability lie in the study of games of chance. Modern probability, however, is a mathematical discipline that has wide applications in a myriad of other mathematical and physical sciences. Drawing on classical gaming examples for motivation, this course will present axiomatic and mathematical aspects of probability. Included will be discussions of random variables (both discrete and continuous), distribution and expectation, independence, laws of large numbers, and the well-known Central Limit Theorem. Many interesting and important applications will also be presented, including some from classical Poisson processes, random walks and Markov Chains.

Requirements/Evaluation: homework, classwork, and exams

Prerequisites: MATH 250 or permission of the instructor

Enrollment Limit: 30

Enrollment Preferences: Priority will be given to Mathematics majors and to Statistics Majors.

Expected Class Size: 20

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:

STAT 341 (D3) MATH 341 (D3)

Quantitative/Formal Reasoning Notes: This is a 300-level Math/Stat course.

Fall 2020
LEC Section: H1  MWF 9:20 am - 10:10 am  Stewart D. Johnson

Spring 2021
LEC Section: H1  TF 3:15 pm - 4:30 pm  Mihai Stoiciu

MATH 350 (F)(S) Real Analysis (QFR)

Why is the product of two negative numbers positive? Why do we depict the real numbers as a line? Why is this line continuous, and what does that actually mean? More fundamentally, what is the definition of a real number? Real analysis addresses such questions, delving into the structure of real numbers and functions on them. Along the way we’ll discuss sequences and limits, series, completeness, compactness, derivatives and integrals, and metric spaces. This course is excellent preparation for graduate studies in mathematics, statistics, and economics. Math 350 and Math 351 will cover the same material for the first part of the course. Math 350 will then delve deeper into the abstract structures of topological and metric spaces, while Math 351 will closely examine some foundational constructs from differential equations, probability, and optimization.

Class Format: Hybrid format. There may be class meetings; remote students will be fully accommodated.

Requirements/Evaluation: homework, classwork, and exams

Prerequisites: MATH 250 or permission of instructor

Enrollment Limit: 20

Enrollment Preferences: Seniors

Expected Class Size: 20
Grading: no pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantative/Formal Reasoning Notes: Math

Fall 2020
LEC Section: R1 MR 3:15 pm - 4:30 pm Leo Goldmakher

Spring 2021
LEC Section: H1 MWF 9:20 am - 10:10 am Stewart D. Johnson

MATH 351 (S) Applied Real Analysis (QFR)
Why is the product of two negative numbers positive? Why do we depict the real numbers as a line? Why is this line continuous, and what does that actually mean? More fundamentally, what is the definition of a real number? Real analysis addresses such questions, delving into the structure of real numbers and functions on them. Along the way we’ll discuss sequences and limits, series, completeness, compactness, derivatives and integrals, and metric spaces. This course is excellent preparation for graduate studies in mathematics, statistics, and economics. Math 350 and Math 351 will cover the same material for the first part of the course. Math 350 will then delve deeper into the abstract structures of topological and metric spaces, while Math 351 will closely examine some foundational constructs from differential equations, probability, and optimization.

Class Format: Hybrid format. There may be class meetings; remote students will be fully accommodated.
Requirements/Evaluation: homework, classwork, and exams
Prerequisites: MATH 250 or permission of the instructor.
Enrollment Limit: 20
Enrollment Preferences: Seniors
Expected Class Size: 20
Grading: no pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantative/Formal Reasoning Notes: Math

Spring 2021
LEC Section: H1 MWF 10:40 am - 11:30 am Stewart D. Johnson

MATH 355 (F)(S) Abstract Algebra (QFR)
Algebra gives us tools to solve equations. The integers, the rationals, and the real numbers have special properties which make algebra work according to the circumstances. In this course, we generalize algebraic processes and the sets upon which they operate in order to better understand, theoretically, when equations can and cannot be solved. We define and study abstract algebraic structures such as groups, rings, and fields, as well as the concepts of factor group, quotient ring, homomorphism, isomorphism, and various types of field extensions. This course introduces students to abstract rigorous mathematics.

Class Format: Format: lecture; Unless circumstances change, students will have the option of taking the course in person or remotely. It is possible that there will be several weeks that are only offered remotely. If taken pass/fail, this course does not count towards the Mathematics major.
Requirements/Evaluation: Problem sets and exams
Prerequisites: MATH 250 or permission of instructor
Enrollment Limit: 20
Enrollment Preferences: Students who have officially declared a major that requires Math 355.
Expected Class Size: 15
Grading: yes pass/fail option, yes fifth course option
Unit Notes: If taken pass/fail, this course does not count towards the Mathematics major.
Distributions: (D3) (QFR)
Quantative/Formal Reasoning Notes: 300-level math course
MATH 361  (F)(S)  Theory of Computation  (QFR)

Cross-listings:  MATH 361  CSCI 361

Secondary Cross-listing

This course introduces a formal framework for investigating both the computability and complexity of problems. We study several models of computation including finite automata, regular languages, context-free grammars, and Turing machines. These models provide a mathematical basis for the study of computability theory—the examination of what problems can be solved and what problems cannot be solved—and the study of complexity theory—the examination of how efficiently problems can be solved. Topics include the halting problem and the P versus NP problem.

Class Format: Lecture content will be delivered through asynchronously viewed video modules. Conference sections meeting twice per week will be used for synchronous discussions. Students should sign up for lecture and one conference section.

Requirements/Evaluation: online multiple choice and short answer questions, weekly problem sets in groups, a research project, and a final examination

Prerequisites:  CSCI 256 or both a 300-level MATH course and permission of instructor

Enrollment Limit:  40(10/con)

Enrollment Preferences:  current or expected Computer Science majors

Expected Class Size:  40

Grading:  yes pass/fail option,  no fifth course option

Distributions:  (D3)  (QFR)

This course is cross-listed and the prefixes carry the following divisional credit:
MATH 361 (D3) CSCI 361 (D3)

Quantative/Formal Reasoning Notes:  This course include regular and substantial problem sets in which quantitative/formal reasoning skills are practiced and evaluated.

Attributes:  COGS Interdepartmental Electives

MATH 368  (S)  Positive Characteristic Commutative Algebra  (QFR)

In commutative algebra, one of the most basic invariants of a ring is its characteristic. This is the smallest multiple of 1 that equals 0. Working over a ring of characteristic zero, versus a ring of characteristic p>0, dramatically changes the proof techniques available to us. This realization has had tremendous consequences in commutative algebra. One of the most useful tools in characteristic p is the Frobenius homomorphism. In this course we
will study several standard notions in commutative algebra, such as regularity of a ring, Cohen-Macaulayness, and being normal and we will see how various "splittings" of the Frobenius allow us to easily detect these properties. Many of these methods are not only applicable to commutative algebra, but also to number theory and algebraic geometry.

Requirements/Evaluation: homework and a final exam
Prerequisites: MATH 355 or permission of instructor
Enrollment Limit: 30
Enrollment Preferences: Math majors primarily, and juniors and seniors secondarily
Expected Class Size: 15
Grading: no pass/fail option, no fifth course option
Distributions: (D3) (QFR)
Not offered current academic year

MATH 374  (S)  Topology  (QFR)
In Real Analysis you learned about metric spaces -- any set of objects endowed with a way of measuring distance -- and the topology of sets in such spaces (open, closed, bounded, etc). In this course we flip this on its head: we explore how to develop analysis (limits, continuity, etc) in spaces where the topology is known but the metric is not. This will lead us to a bizarre and fascinating version of geometry in which we cannot distinguish between shapes that can be continuously deformed into one another. Not only does this theory turn out to be beautiful in the abstract, it has become a vital part of data analysis and is also connected to many areas of math and physics. This course is excellent preparation for graduate programs in mathematics.

Class Format: Taught remotely, but synchronously. While recordings of lectures will be made available, all participants are expected to make their best effort to attend the class over Zoom. In addition to class meetings, there will be tutorial sessions with a TA once per week.
Requirements/Evaluation: homework, tutorials, and exams
Prerequisites: MATH 350 or 351; not open to students who have taken MATH 323. Familiarity with basic group theory recommended, but not required.
Enrollment Limit: 20
Enrollment Preferences: Juniors and seniors
Expected Class Size: 20
Grading: no pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: It's math.

Spring 2021
LEC Section: R1  MR 3:15 pm - 4:30 pm  Leo  Goldmakher

MATH 390  Undergraduate Research Topics in Algebra  (QFR)
The well-known trace map on matrices can be generalized to a map on other algebraic objects. Undergraduates, graduates students and experts in Representation Theory, Commutative Algebra and Algebraic Geometry have been driving recent developments in the theory of trace modules and finding exciting new applications in all of these fields. This course will serve as an introduction to mathematical research with the aim of producing original research in modern trace theory. Students in this tutorial will read and synthesize research papers, discuss the formation of research questions in pure mathematics, and engage in original mathematical research.

Requirements/Evaluation: oral presentations; writing assignments (summarizing papers, reflections on mathematical research, original research); participation in the course project
Prerequisites: Math 355
Enrollment Limit: 10
Enrollment Preferences: Juniors and Seniors
Expected Class Size: 7
Grading:

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: This is post-core math class; students will be required to produce mathematical proofs.

Not offered current academic year

MATH 391 (F) Introduction to computer algebra (QFR)
Students will learn new mathematics in the context of computer-based exposition, experimentation, and interaction. They will gain proficiency with Sage, GAP, Macaulay2, or Mathematica, and possibly one of the more-specialized systems SnapPea, kenzo, magma, MATLAB, Perseus, coq, etc. Individuals and teams will build interactive demonstrations of mathematical theorems, which will then be appreciated by the instructor and the rest of the class. No prior programming experience is expected.

Class Format: Class will be held online, but there will be recorded components, asynchronous interactive components, and outside-of-class small-group online meetings.

Requirements/Evaluation: exams, homework, projects

Prerequisites: Math 355 or permission of instructor

Enrollment Limit: 20

Enrollment Preferences: math majors

Expected Class Size: 15

Grading: yes pass/fail option, no fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: Mathematical programming requires complete synthesis of abstract concepts to produce computer code, which is necessarily formal.

Fall 2020
LEC Section: R1 TR 6:45 pm - 8:00 pm John D. Wiltshire-Gordon

MATH 392 (S) Undergraduate Research Topics in Graph Theory (WS) (QFR)
Graph theory is a vibrant area of research with many applications to the social sciences, psychology, and economics. In this project-based tutorial, students will select among the presented topics and will develop research questions and undertake original research in the field. Student assessment is based on drafts of research project manuscript and presentations.

Requirements/Evaluation: presentations and written project manuscript

Prerequisites: MATH 355 or permission of the instructor

Enrollment Limit: 10

Enrollment Preferences: programming experience, students with interests in the intersection of combinatorics and graph theory

Expected Class Size: 10

Grading: no pass/fail option, no fifth course option

Distributions: (D3) (WS) (QFR)

Writing Skills Notes: This course will require multiple revisions of a manuscript related to the research project at hand. The final result will be a 10-20 page research article and the course will be designed as a writing intensive course.

Quantitative/Formal Reasoning Notes: The course deals with mathematical research in graph theory and is a quantitative and formal reasoning course.

Spring 2021
TUT Section: RT1 TBA Pamela E. Harris

MATH 397 (F)(S) Independent Study: Mathematics
Directed 300-level independent study in Mathematics.
Prerequisites: permission of department

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3)

---

Fall 2020

IND Section: H1  TBA  Mihai Stoiciu

Spring 2021

IND Section: R1  Cancelled

---

MATH 398 (S) Independent Study: Mathematics

Directed 300-level independent study in Mathematics.

Prerequisites: permission of department

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3)

---

Spring 2021

IND Section: R1  TBA  Mihai Stoiciu

---

MATH 402 (F) Measure Theory and Hilbert Spaces (QFR)

How large is the unit square? One might measure the number of individual points in the square (uncountably infinite), the area of the square (1), or the dimension of the square (2). But what about for more complicated sets, e.g., the set of all rational points in the unit square? What's the area of this set? What's the dimension? In this course we'll come up with precise ways to measure size -- length, area, volume, dimension -- that apply to a broad array of sets. Along the way we'll encounter Lebesgue measure and Lebesgue integration, Hausdorff measure and fractals, space-filling curves and the Banach-Tarski paradox. We will also investigate Hilbert spaces, mathematical objects that combine the tidiness of linear algebra with the power of analysis and are fundamental to the study of differential equations, functional analysis, harmonic analysis, and ergodic theory, and also apply to fields like quantum mechanics and machine learning. This material provides good preparation for graduate studies in mathematics, statistics and economics.

Class Format: Discussion-based course held remotely.

Requirements/Evaluation: performance on homework assignments and exams

Prerequisites: MATH 350 or MATH 351 or permission of instructor

Enrollment Limit: 30

Enrollment Preferences: Seniors

Expected Class Size: 20

Grading: no pass/fail option, yes course option

Distributions: (D3) (QFR)

Quantative/Formal Reasoning Notes: Math

---

Fall 2020

LEC Section: R1  TF 3:15 pm - 4:30 pm  Leo Goldmakher

---

MATH 403 (S) Measure and Ergodic Theory (QFR)

An introduction to measure theory and ergodic theory. Measure theory is a generalization of the notion of length and area, has been used in the study of stochastic (probabilistic) systems. The course covers the construction of Lebesgue and Borel measures, measurable functions, and Lebesgue integration. Ergodic theory studies the probabilistic behavior of dynamical systems as they evolve through time, and is based on measure theory. The course will cover basic notions, such as ergodic transformations, weak mixing, mixing, and Bernoulli transformations, and transformations admitting
and not admitting an invariant measure. There will be an emphasis on specific examples such as group rotations, the binary odometer transformations, and rank-one constructions. The Ergodic Theorem will also be covered, and will be used to illustrate notions and theorems from measure theory.

Requirements/Evaluation: homework and exams
Prerequisites: MATH 350 or MATH 351 or permission of instructor
Enrollment Limit: 25
Enrollment Preferences: Mathematics majors
Expected Class Size: 15-20
Grading: yes pass/fail option, yes fifth course option
Unit Notes: senior major course
Distributions: (D3) (QFR)

Not offered current academic year

MATH 404 (F) Random Matrix Theory (QFR)
Initiated by research in multivariate statistics and nuclear physics, the study of random matrices is nowadays an active and exciting area of mathematics, with numerous applications to theoretical physics, number theory, functional analysis, optimal control, and finance. Random Matrix Theory provides understanding of various properties (most notably, statistics of eigenvalues) of matrices with random coefficients. This course will provide an introduction to the basic theory of random matrices, starting with a quick review of Linear Algebra and Probability Theory. We will continue with the study of Wigner matrices and prove the celebrated Wigner's Semicircle Law, which brings together important ideas from analysis and combinatorics. After this, we will turn our attention to Gaussian ensembles and investigate the Gaussian Orthogonal Ensemble (GOE) and the Gaussian Unitary Ensemble (GUE). The last lectures of the course will be dedicated to random Schrodinger operators and their spectral properties (in particular, the phenomenon called Anderson localization). Applications of Random Matrix Theory to theoretical physics, number theory, statistics, and finance will be discussed throughout the semester.

Requirements/Evaluation: homework assignments and exams
Prerequisites: experience with Real Analysis (MATH 350 or MATH 351) and with Probability (MATH 341 or STAT 201)
Enrollment Limit: 40
Enrollment Preferences: Mathematics and Statistics majors
Expected Class Size: 20
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3) (QFR)
Quantitative/Formal Reasoning Notes: This course expands ideas in probability and statistics from random variables (1x1 random matrices) to nxn random matrices. The students will learn to model complex physical phenomena using random matrices and study them using rigorous mathematical tools and concepts.

Not offered current academic year

MATH 407 (F) Dance of the Primes (QFR)
Prime numbers are the building blocks for all numbers and hence for most of mathematics. Though there are an infinite number of them, how they are spread out among the integers is still quite a mystery. Even more mysterious and surprising is that the current tools for investigating prime numbers involve the study of infinite series. Function theory tells us about the primes. We will be studying one of the most amazing functions known: the Riemann Zeta Function. Finding where this function is equal to zero is the Riemann Hypothesis and is one of the great, if not greatest, open problems in mathematics. Somehow where these zeros occur is linked to the distribution of primes. We will be concerned with why anyone would care about this conjecture. More crassly, why should solving the Riemann Hypothesis be worth one million dollars? (Which is what you will get if you solve it, beyond the eternal fame and glory.)

Requirements/Evaluation: exams and weekly homework assignments
Prerequisites: MATH 350 or MATH 351, and MATH 355
Enrollment Limit: 30
Enrollment Preferences: seniors
Expected Class Size: 10
Optimal packing problems arise in many important problems, and have been a source of excellent mathematics for centuries. The Kepler Problem (what is the most efficient way to pack balls in three-space) is a good example. The original formulation has been used in such diverse areas as stacking cannonballs on battleships to grocers preparing fruit displays, and its generalizations allow the creation of powerful error detection and correction codes. While the solution of the Kepler Problem is now known, the higher dimensional version is very much open. There has been remarkable progress in the last few years, with number theory playing a key role in these results. We will develop sufficient background material to understand many of these problems and the current state of the field. Pre-requisites are real analysis.

**Requirements/Evaluation:** Class participation, homework, exams and participation in writing a textbook on the material. Each student will be responsible for working on a chapter of a book based on this material. In addition to obtaining critical writing feedback from myself and my co-author (who is a world expert in the subject), depending on timing we will also be able to share comments from an editor of a major publishing house or a referee. Chapters can range from short snapshots of a subject, on the order of 5 pages, to longer technical derivations of perhaps 10-30 pages.

**Prerequisites:** Math 350 or 351

**Enrollment Limit:** 40

**Expected Class Size:** 20

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This is a 400 level math class

---

**MATH 411 (S) Commutative Algebra** (QFR)

Commutative Algebra is an essential area of mathematics that provides indispensable tools to many areas, including Number Theory and Algebraic Geometry. This course will introduce you to the fundamental concepts for the study of commutative rings, with a special focus on the notion of "prime ideals," and how they generalize the well-known notion of primality in the set of integers. Possible topics include Noetherian rings, primary decomposition, localizations and quotients, height, dimension, basic module theory, and the Krull Altitude Theorem.

**Requirements/Evaluation:** homework and exams

**Prerequisites:** MATH 355 or permission of instructor

**Enrollment Limit:** 30

**Expected Class Size:** 15

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Not offered current academic year**

---

**MATH 419 (S) Algebraic Number Theory** (QFR)

We all know that integers can be factored into prime numbers and that this factorization is essentially unique. In more general settings, it often still makes sense to factor numbers into "primes," but the factorization is not necessarily unique! This surprising fact was the downfall of Lamé's attempted proof of Fermat's Last Theorem in 1847. Although a valid proof was not discovered until over 150 years later, this error gave rise to a new branch of mathematics: algebraic number theory. In this course, we will study factorization and other number-theoretic notions in more abstract algebraic settings, and we will see a beautiful interplay between groups, rings, and fields.

**Requirements/Evaluation:** homework assignments and exams

---

**MATH 408 (F) L-Functions and Sphere Packing** (QFR)

Optimal packing problems arise in many important problems, and have been a source of excellent mathematics for centuries. The Kepler Problem (what is the most efficient way to pack balls in three-space) is a good example. The original formulation has been used in such diverse areas as stacking cannonballs on battleships to grocers preparing fruit displays, and its generalizations allow the creation of powerful error detection and correction codes. While the solution of the Kepler Problem is now known, the higher dimensional version is very much open. There has been remarkable progress in the last few years, with number theory playing a key role in these results. We will develop sufficient background material to understand many of these problems and the current state of the field. Pre-requisites are real analysis.

**Requirements/Evaluation:** Class participation, homework, exams and participation in writing a textbook on the material. Each student will be responsible for working on a chapter of a book based on this material. In addition to obtaining critical writing feedback from myself and my co-author (who is a world expert in the subject), depending on timing we will also be able to share comments from an editor of a major publishing house or a referee. Chapters can range from short snapshots of a subject, on the order of 5 pages, to longer technical derivations of perhaps 10-30 pages.

**Prerequisites:** Math 350 or 351

**Enrollment Limit:** 40

**Expected Class Size:** 20

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** It is a math course.

**Not offered current academic year**

---

**Fall 2020**

**LEC Section:** H1 MWF 12:00 pm - 12:50 pm Steven J. Miller
Prerequisites: MATH 355, or permission of instructor

Enrollment Limit: 25

Expected Class Size: 20

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Not offered current academic year

MATH 420 (S) Analytic Number Theory (QFR)

How many primes are smaller than x? How many divisors does an integer n have? How many different numbers appear in the N x N multiplication table? Precise formulas for these quantities probably don't exist, but over the past 150 years tremendous progress has been made towards understanding these and similar questions using tools and methods from analysis. The goal of this tutorial is to explain and motivate the ubiquitous appearance of analysis in modern number theory—a surprising fact, given that analysis is concerned with continuous functions, while number theory is concerned with discrete objects (integers, primes, divisors, etc). Topics to be covered will include some subset of the following: asymptotic analysis, partial and Euler-Maclaurin summation, counting divisors and Dirichlet's hyperbola method, the randomness of prime factorization and the Erdos-Kac theorem, the partition function and the saddle point method, the prime number theorem and the Riemann zeta function, primes in arithmetic progressions and Dirichlet L-functions, the Goldbach conjecture and the circle method, and sieve methods and gaps between primes.

Requirements/Evaluation: Regularly preparing lectures and writing expository essays in LaTeX. No exams.

Prerequisites: MATH 350 or MATH 351 and familiarity with basic modular arithmetic are hard prerequisites. Familiarity with complex analysis and abstract algebra recommended, but not required.

Enrollment Limit: 10

Enrollment Preferences: Students with complex analysis background will be given priority.

Expected Class Size: 10

Grading: no pass/fail option, no fifth course option

Distributions: (D3) (QFR)

Quantitative/Formal Reasoning Notes: It's math.

Spring 2021

TUT Section: RT1 TBA Leo Goldmakher

MATH 421 (S) Quandles, Knots and Virtual Knots (QFR)

A quandle is an algebraic object that, like a group, has a "multiplication" of pairs of elements that satisfies certain axioms. But the quandle axioms are very different from the group axioms, and quandles turn out to be incredibly useful when considering the mathematical theory of knots. In this course, we will learn about this relatively new area of research (1982) and learn some knot theory and see how quandles apply to both classical knot theory and the relatively new area of virtual knot theory (1999).

Requirements/Evaluation: problem sets, tests, and a 3-page paper

Prerequisites: MATH 355

Enrollment Limit: 40

Enrollment Preferences: discretion of the instructor

Expected Class Size: 15

Grading: no pass/fail option, no fifth course option

Distributions: (D3) (QFR)

Not offered current academic year

MATH 422 (F) Algebraic Topology (QFR)

Is a sphere really different from a torus? Can a sphere be continuously deformed to a point? Algebraic Topology concerns itself with the classification and study of topological spaces via algebraic methods. The key question is this: How do we really know when two spaces are different and in what
senses can we claim they are the same? Our answer will use several algebraic tools such as groups and their normal subgroups. In this course we will develop several notions of "equality" starting with the existence of homeomorphisms between spaces. We will then explore several weakenings of this notion, such as homotopy equivalence, having isomorphic homology or fundamental groups, and having homeomorphic universal covers.

**Requirements/Evaluation:** homework and exams  
**Prerequisites:** MATH 355 or permission of instructor  
**Enrollment Limit:** 30  
**Enrollment Preferences:** Math majors primarily, the juniors and seniors  
**Expected Class Size:** 15  
**Grading:** no pass/fail option, no fifth course option  
**Distributions:** (D3) (QFR)

MATH 426 (F) Differential Topology (QFR)

Differential topology marries the rubber-like deformations of topology with the computational exactness of calculus. This subfield of mathematics asks and answers questions like "Can you take an integral on the surface of doughnut?" and includes far-reaching applications in relativity and robotics. This tutorial will provide an elementary and intuitive introduction to differential topology. We will begin with the definition of a manifold and end with a generalized understanding of Stokes Theorem.

**Requirements/Evaluation:** weekly homework, weekly presentations, and final paper  
**Prerequisites:** MATH 350 (students who have not taken MATH 250 may enroll only with permission of the instructor)  
**Enrollment Limit:** 10  
**Enrollment Preferences:** seniors, majors  
**Expected Class Size:** 10  
**Grading:** no pass/fail option, no fifth course option  
**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** There will be weekly math problem sets.

Not offered current academic year

MATH 427 (S) Tiling Theory (QFR)

Since humans first used stones and bricks to tile the floors of their domiciles, tiling has been an area of interest. Practitioners include artists, engineers, designers, architects, crystallographers, scientists and mathematicians. This course will be an investigation into the mathematical theory of tiling. The course will focus on tilings of the plane, including topics such as the symmetry groups of tilings, the topology of tilings, the ergodic theory of tilings, the classification of tilings and the aperiodic Penrose tilings. We will also look at tilings in higher dimensions, including "knotted tilings".

**Requirements/Evaluation:** problem assignments, exams and a presentation/paper  
**Prerequisites:** MATH 250 Linear Algebra and MATH 355 Abstract Algebra  
**Enrollment Limit:** 30  
**Enrollment Preferences:** senior majors, seniors, juniors, sophomores, first-year students (this is a senior seminar, one of which is required for all senior majors, so they have first preference)  
**Expected Class Size:** 20  
**Grading:** no pass/fail option, yes fifth course option  
**Materials/Lab Fee:** cost of book which will be under $50  
**Distributions:** (D3) (QFR)

Not offered current academic year

MATH 428 (S) Catching Robbers and Spreading Information (QFR)

Cops and robbers is a widely studied game played on graphs that has connections to searching algorithms on networks. The cop number of a graph is the smallest number of cops needed to guarantee that the cops can catch a robber in the graph. Similar combinatorial games such as "zero forcing"
can be used to model the spread of information. The idea of "throttling" is to spread the information (or catch the robber) as efficiently as possible. This course will survey some of the main results about cops and robbers and the cop number. We will also explore recent research on throttling for cops and robbers, zero forcing, and other variants.

**Class Format:** interactive activities and discussion

**Requirements/Evaluation:** problem sets, investigation journal, final presentation

**Prerequisites:** MATH 200 and MATH 355

**Enrollment Limit:** 25

**Enrollment Preferences:** seniors

**Expected Class Size:** 20

**Grading:** yes pass/fail option, no fifth course option

**Distributions:** (D3) (QFR)

**Quantative/Formal Reasoning Notes:** The course will involve mathematical proofs.

Not offered current academic year

MATH 433  (S)  Mathematical Modeling  (QFR)

Mathematical modeling means (1) translating a real-life problem into a mathematical object, (2) studying that object using mathematical techniques, and (3) interpreting the results in order to learn something about the real-life problem. Mathematical modeling is used in biology, economics, chemistry, geology, sociology, political science, art, and countless other fields. This is an advanced, seminar-style, course appropriate for students who have strong enthusiasm for applied mathematics, data science, and collaborative teamwork.

**Class Format:** To afford students flexibility during the COVID pandemic, this course is taught online, largely asynchronously. There is no lecture component. Students will read research literature, work on structured and open-ended projects, and participate in synchronous small-group meetings with the instructor via videoconference. The vast majority of work in this course requires students to collaborate with each other.

**Requirements/Evaluation:** Students will complete reading assignments, writing assignments, modeling activities, research projects, and will record several presentations to be shared with the rest of the class. To move towards a non-hierarchical, transparent, and egalitarian grading system, the instructor follows the policy of "ungrading." Over the course of the semester, students will develop a rubric to assess their own learning and will evaluate themselves according to this rubric.

**Prerequisites:** MATH 250; MATH 309 or similar; and some experience with computer programming (equivalent to CSCI 134 or MATH 307).

**Enrollment Limit:** 20

**Enrollment Preferences:** Professor's discretion

**Expected Class Size:** 20

**Grading:** yes pass/fail option, yes fifth course option

**Distributions:** (D3) (QFR)

**Quantative/Formal Reasoning Notes:** This course focuses substantially on using mathematical and statistical tools and frameworks to describe, predict, and understand real-world systems.

Spring 2021

SEM Section: R1    TR 11:30 am - 12:45 pm    Chad M. Topaz

MATH 434  (F)  Applied Dynamics and Optimal Control  (QFR)

We seek to understand how dynamical systems evolve, how that evolution depends on the various parameters of the system, and how we might manipulate those parameters to optimize an outcome. We will explore the language of dynamics by deepening our understanding of differential and difference equations, study parameter dependence and bifurcations, and explore optimal control through Pontryagin’s maximum principle and Hamilton-Jacobi-Bellman equations. These tools have broad application in ecology, economics, finance, and engineering, and we will draw on basic models from these fields to motivate our study.

**Requirements/Evaluation:** exams and homework assignments

**Prerequisites:** MATH 209 or PHYS 210, and MATH 350 or 351, or permission of the instructor

**Enrollment Limit:** 20
Enrollment Preferences: Preference will be given to senior math majors.

Expected Class Size: 20

Grading: yes pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantative/Formal Reasoning Notes: This is a 400 level math course.

Fall 2020

LEC Section: H1 MWF 10:40 am - 11:30 am Stewart D. Johnson

MATH 453 (F) Partial Differential Equations (QFR)
Partial differential equations (PDE) arise as mathematical models of phenomena in chemistry, ecology, economics, electromagnetics, fluid dynamics, neuroscience, thermodynamics, and more. We introduce PDE models and develop techniques for studying them. Topics include: derivation, classification, and physical interpretation of canonical PDE; solution techniques, including separation of variables, series solutions, integral transforms, and characteristics; and application to problems in the natural and social sciences.

Requirements/Evaluation: quizzes/exams, problem sets, projects and activities
Prerequisites: MATH 150-151; MATH 209 or MATH/PHYS 210 or MATH 309
Enrollment Limit: 30

Enrollment Preferences: Professor's discretion

Expected Class Size: 25

Grading: no pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Not offered current academic year

MATH 456 (F) Representation Theory (QFR)
Representation theory has applications in fields such as physics (via models for elementary particles), engineering (considering symmetries of structures), and even in voting theory (voting for committees in agreeable societies). This course will introduce the concepts and techniques of the representation theory of finite groups, and will focus on the representation theory of the symmetric group. We will undertake this study through a variety of perspectives, including general representation theory, combinatorial algorithms, and symmetric functions.

Requirements/Evaluation: evaluation will be based primarily on homework, in class presentations, and exams
Prerequisites: MATH 355
Enrollment Limit: 40

Enrollment Preferences: junior and senior Math majors

Expected Class Size: 15

Grading: no pass/fail option, yes fifth course option

Distributions: (D3) (QFR)

Quantative/Formal Reasoning Notes: This is a 400-level Math course.

Fall 2020

LEC Section: R1 TF 1:30 pm - 2:45 pm John D. Wiltshire-Gordon

MATH 458 (S) Algebraic Combinatorics (QFR)
Algebraic combinatorics is a branch of mathematics at the intersection of combinatorics and algebra. On the one hand, we study combinatorial structures using algebraic techniques, while on the other we use combinatorial arguments and methods to solve problems in algebra. This course will focus on the study of symmetric functions, young tableaux, matroids, graph theory, and other related topics.

Requirements/Evaluation: homework assignments, proof portfolio, individual and group projects
MATH 459  (S)  Applied Partial Differential Equations  (QFR)

Partial differential equations (PDE) arise as mathematical models of phenomena in chemistry, ecology, economics, electromagnetics, epidemiology, fluid dynamics, neuroscience, and much more. Furthermore, the study of partial differential equations connects with diverse branches of mathematics including analysis, geometry, algebra, and computation. Adopting an applied viewpoint, we develop techniques for studying PDE. We draw from a body of knowledge spanning classic work from the time of Isaac Newton right up to today's cutting edge applied mathematics research. This tutorial is appropriate as a second course in differential equations. In this tutorial, students will: build and utilize PDE-based models; determine the most appropriate tools to apply to a PDE; apply the aforementioned tools; be comfortable with open-ended scientific work; read applied mathematical literature; communicate applied mathematics clearly, precisely, and appropriately; collaborate effectively.

Requirements/Evaluation:  participation, problem sets, oral presentations, oral exams, and a final project

Prerequisites:  MATH 209 or MATH/PHYS 210 or MATH 309 or permission of instructor; students who have taken MATH 453 may not enroll in MATH 458 without permission of the instructor

Enrollment Limit:  10

Enrollment Preferences:  students with an interest in applied mathematics, selected to create a diverse set of tutorial participants

Expected Class Size:  10

Grading:  no pass/fail option,     no fifth course option

Distributions:  (D3)  (QFR)

Quantative/Formal Reasoning Notes:  This tutorial involves regular and substantial problem sets in which quantitative/formal reasoning skills are practiced and evaluated.

Not offered current academic year

MATH 466  (F)  Advanced Applied Analysis  (QFR)

This course further develops and explores topics and concepts from real analysis, with special emphasis on introducing students to subject matter and techniques that are useful for graduate study in mathematics or an allied field, as well as applications in industry. Topics include Benford's law of digit bias, random matrix theory, and Fourier analysis, and as time permits additional areas based on student interest from analytic number theory, generating functions and probabilistic methods. This will be an intense, fast paced class which will give a flavor for graduate school. In addition to standard homework problems, students will assist in writing both reviews for MathSciNet and referee reports for papers for journals, write programs to investigate and conjecture, and read classic and current research papers, and possibly apply these and related methods to real world problems.

Requirements/Evaluation:  homework, exams, possible paper/presentation

Prerequisites:  MATH 350 or MATH 351

Enrollment Limit:  40

Enrollment Preferences:  discretion of the instructor

Expected Class Size:  30

Grading:  yes pass/fail option,     yes fifth course option

Distributions:  (D3)  (QFR)

Not offered current academic year

MATH 474  (S)  Tropical Geometry  (QFR)
This course offers an introduction to tropical geometry, a young subject that has already established deep connections between itself and pure and applied mathematics. We will study a rich variety of objects arising from polynomials over the min-plus semiring, where addition is defined as taking a minimum, and multiplication is defined as usual addition. We will learn how these polyhedral objects connect to other areas of mathematics like algebraic geometry, and how they can be applied to solve problems in scheduling theory, phylogenetics, and other diverse fields.

**Class Format:** Hybrid; if possible we will have classes in person, with remote students attending via Zoom.

**Requirements/Evaluation:** participation, problem sets, quizzes, exams, and a final project

**Prerequisites:** MATH 355 or permission of instructor

**Enrollment Limit:** 25

**Enrollment Preferences:** Senior math majors

**Expected Class Size:** 15

**Grading:** no pass/fail option, no fifth course option

**Distributions:** (D3) (QFR)

**Quantitative/Formal Reasoning Notes:** This course builds quantitative skills

Spring 2021

LEC Section: H1    TR 9:45 am - 11:00 am    Ralph E. Morrison

**MATH 482 (F) Homological Algebra** (QFR)

Though a relatively young subfield of mathematics, Homological Algebra has earned its place by supplying powerful tools to solve questions in the much older fields of Commutative Algebra, Algebraic Geometry and Representation Theory. This class will introduce theorems and tools of Homological Algebra, grounding its results in applications to polynomial rings and their quotients. We will focus on some early groundbreaking results and learn some of Homological Algebra's most-used constructions. Possible topics include tensor products, chain complexes, homology, Ext, Tor and Hilbert's Syzygy Theorem.

**Requirements/Evaluation:** homework and exams

**Prerequisites:** MATH 355

**Enrollment Limit:** 20

**Enrollment Preferences:** junior and senior math majors

**Expected Class Size:** 12

**Grading:** no pass/fail option, yes fifth course option

**Unit Notes:** this course is not a senior seminar, so it does not fulfill the senior seminar requirement for the Math major

**Distributions:** (D3) (QFR)

Not offered current academic year

**MATH 484 (S) Galois Theory** (QFR)

Some equations--such as \(x^5 - 1 = 0\)--are easy to solve. Others--such as \(x^5 - x - 1 = 0\)--are very hard, if not impossible (using standard mathematical operations). Galois discovered a deep connection between field theory and group theory that led to a criterion for checking whether or not a given polynomial can be easily solved. His discovery also led to many other breakthroughs, for example proving the impossibility of squaring the circle or trisecting a typical angle using compass and straightedge. From these not-so-humble beginnings, Galois theory has become a fundamental concept in modern mathematics, from topology to number theory. In this course we will develop the theory and explore its applications to other areas of math.

**Requirements/Evaluation:** written homeworks, oral presentations, and exams

**Prerequisites:** MATH 355

**Enrollment Limit:** 15

**Enrollment Preferences:** discretion of the instructor

**Expected Class Size:** 10

**Grading:** no pass/fail option, yes fifth course option

**Unit Notes:** this course is not a senior seminar, so it does not fulfill the senior seminar requirement for the Math major
MATH 485  (F) Complex Analysis  (QFR)
The calculus of complex-valued functions turns out to have unexpected simplicity and power. As an example of simplicity, every complex-differentiable function is automatically infinitely differentiable. As examples of power, the so-called residue calculus permits the computation of impossible integrals, and conformal mapping reduces physical problems on very general domains to problems on the round disc. The easiest proof of the Fundamental Theorem of Algebra, not to mention the first proof of the Prime Number Theorem, used complex analysis.

Requirements/Evaluation:  homework, classwork, and exams
Prerequisites:  MATH 350 or MATH 351 or permission of instructor
Enrollment Limit:  40
Expected Class Size:  30
Grading:  yes pass/fail option,  yes fifth course option

Unit Notes:  this course is not a senior seminar, so it does not fulfill the senior seminar requirement for the Math major
Distributions:  (D3)  (QFR)

Quantative/Formal Reasoning Notes:  Advanced mathematics course with weekly or daily problem sets.

Not offered current academic year

MATH 487  (S) Computational Algebraic Geometry  (QFR)
Algebraic geometry is the study of shapes described by polynomial equations. It has been a major part of mathematics for at least the past two hundred years, and has influenced a tremendous amount of modern mathematics, ranging from number theory to robotics. In this course, we will develop the Ideal-Variety Correspondence that ties geometric shapes to abstract algebra, and will use computational tools to explore this theory in a very explicit way.

Requirements/Evaluation:  homework, exams, and final project
Prerequisites:  MATH 355
Enrollment Limit:  40
Enrollment Preferences:  instructor decision
Expected Class Size:  15
Grading:  no pass/fail option,  yes fifth course option

Unit Notes:  this course is not a senior seminar, so it does not fulfill the senior seminar requirement for the Math major
Distributions:  (D3)  (QFR)

Not offered current academic year

MATH 493  (F) Senior Honors Thesis: Mathematics
Mathematics senior honors thesis; this is part of a full-year thesis (493-494). Each student carries out an individual research project under the direction of a faculty member that culminates in a thesis. See description under The Degree with Honors in Mathematics.

Grading:  yes pass/fail option,  yes fifth course option
Distributions:  (D3)

Fall 2020
HON Section:  H1  TBA  Mihai Stoiciu

MATH 494  (S) Senior Honors Thesis: Mathematics
Mathematics senior honors thesis; this is part of a full-year thesis (493-494). Each student carries out an individual research project under the direction of a faculty member that culminates in a thesis. See description under The Degree with Honors in Mathematics.
Grading: yes pass/fail option, yes fifth course option
Distributions: (D3)

Spring 2021
HON Section: R1   TBA   Mihai Stoiciu

**MATH 497 (F) Independent Study: Mathematics**
Directed 400-level independent study in Mathematics.

**Prerequisites:** permission of department

**Grading:** yes pass/fail option, yes fifth course option
**Distributions:** (D3)

Fall 2020
IND Section: H1   TBA   Mihai Stoiciu

**MATH 498 (S) Independent Study: Mathematics**
Directed 400-level independent study in Mathematics.

**Prerequisites:** permission of department

**Grading:** yes pass/fail option, yes fifth course option
**Distributions:** (D3)

Spring 2021
IND Section: R1   TBA   Mihai Stoiciu

**MATH 499 (F)(S) Senior Colloquium**
Mathematics senior colloquium. Meets every week for two hours both fall and spring. Senior majors must participate at least one hour a week. This colloquium is in addition to the regular four semester-courses taken by all students.

**Class Format:** colloquium

**Grading:** non-graded
**Distributions:** No divisional credit

Fall 2020
LEC Section: H1   TBA   Mihai Stoiciu

Spring 2021
LEC Section: R1   MTW 1:30 pm - 2:45 pm   Mihai Stoiciu

Winter Study ---------------------------------------------------------------

**MATH 30 (W) Senior Project: Mathematics**
To be taken by candidates for honors in Mathematics other than by thesis route.

**Class Format:** honors project

**Grading:** pass/fail only

*Not offered current academic year*
MATH 31 (W) Senior Thesis: Mathematics
To be taken by students registered for Mathematics 493-494.

Class Format: thesis
Grading: pass/fail only
Not offered current academic year

MATH 41 (W) Introduction to Data Science - Intensive
Data science brings together techniques from computing, mathematics, and statistics to extract knowledge from data in fields of application as diverse as climate science, particle physics, electoral politics, literary analysis, and countless others. This course provides an introduction to data science techniques. First, using the computational package R, students will learn how to acquire, clean, explore, summarize, visualize, and communicate data. Second, in a series of nontechnical guest lectures, professional data scientists will share the types of work they do. Finally, students will carry out a small project, applying their data science skills to problems that interest them. This course requires no background in computer programming, mathematics, or statistics. However, this is an intensive course, and students must have enthusiasm to learn programming and a willingness to practice this skill for several hours per day.

Class Format: To afford students flexibility during the COVID pandemic, this course is taught online. Students will watch videos and complete data science modules asynchronously, and will participate in occasional synchronous lectures.

Requirements/Evaluation: Students will complete data science modules and a final project. To move towards a non-hierarchical, transparent, and egalitarian grading system, the instructor follows the policy of "ungrading." Over the course of the term, students will develop a rubric to assess their own learning and will evaluate themselves according to this rubric.

Prerequisites: Willingness to learn a new field; willingness to practice computer programming intensively.

Enrollment Limit: 20
Enrollment Preferences: Contact the Office of the Dean of the College.

Expected Class Size: 20
Grading: pass/fail only

Unit Notes: This course is designed to count for both full semester, Winter Study, and QFR credit. Once a dean approves enrollment, the Registrar’s Office will register students in both MATH 101 and MATH 41.

Winter 2021
SEM Section: R1    TBA    Chad M. Topaz

MATH 99 (W) Independent Study: Mathematics
Open to upperclass students. Students interested in doing an independent project (99) during Winter Study must make prior arrangements with a faculty sponsor. The student and professor then complete the independent study proposal form available online. The deadline is typically in late September. Proposals are reviewed by the pertinent department and the Winter Study Committee. Students will be notified if their proposal is approved prior to the Winter Study registration period.

Class Format: independent study
Grading: pass/fail only
Not offered current academic year